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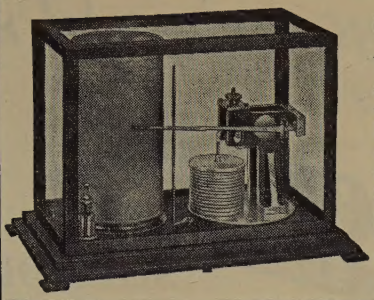

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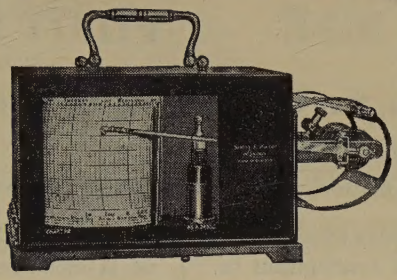

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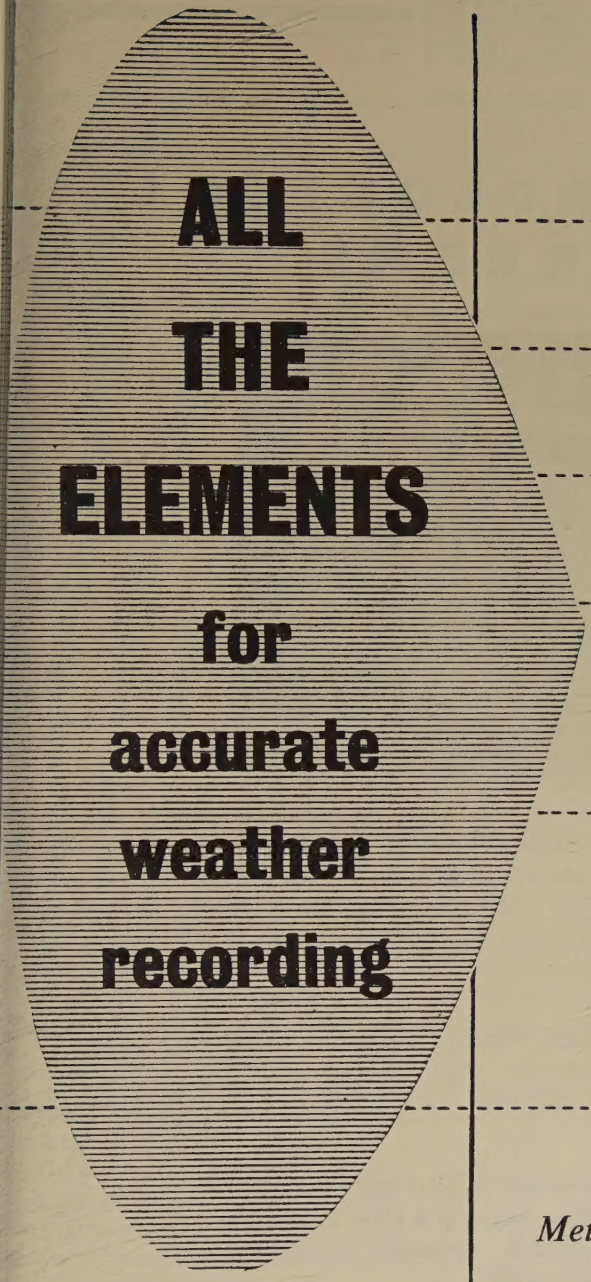

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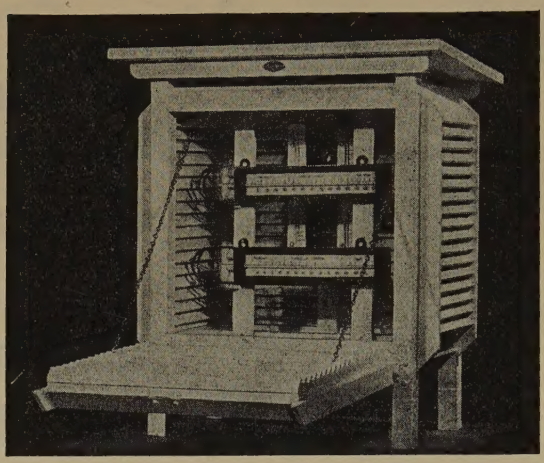

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Some measurements on undercutting cold air

By F. A. BERSON

Division of Meteorological Physics, Commonwealth Scientific and Industrial Research Organization, Melbourne, Australia.

(Manuscript received 11 March 1957, in revised form 12 August 1957)

SUMMARY

From time series of closely-spaced upper-wind (pilot-balloon) measurements in the rear of synoptic es of wind shift and cooling, the configuration of the elevated wind-shear layer and the strength of the flow neath are deduced. The height of the shear layer does not rise uniformly with time, but at one stage mains constant or decreases, before resuming a sustained rise. It is demonstrated by the method of namic similarity that this phenomenon is broadly the counterpart of the friction head on density surges in ratified liquids.

Cumulative inflow into the advancing wedge of cold air is compared with the total mass in the cold air dge and found to be generally in excess of this, the exceptions occurring with sea-breezes or analogous enomena on a larger scale (coastal fronts). In one case where sufficient observations are available, vertical locities are computed. The excess inflow is shown to be associated with a vertical circulation involving nsiderable mass transfer upward across the shear zone.

1. INTRODUCTION

A basic problem in frontology is to deduce the steady-state shape which a mass of old air assumes when it breaks into warmer air at rest or in uniform motion. Margules's rmula for the slope of the interface is valid only when surface drag, internal friction nd acceleration can be disregarded.

For a decade or so, little has been added to the treatment of the problem by v. Kármán 940) and Prandtl (1952). Perhaps this is not surprising as during this period the classical ontal concept was abandoned as inadequate to serve as a synoptic model for the wind-ift line of the sub-tropics, the easterly waves of the tropics and the (prefrontal) squall ne of middle latitudes.

In Australia new approaches were made to the problem of disturbances in the tropical sterlies (Gibbs 1945) and of the meridional front extending from troughs in the westerlies almer 1942; Gibbs, Gotley and Martin 1952). It was shown in a recent aerological udy of the winter meridional front in Western Australia (Troup 1956) that disturbances ith marked windshift at the surface and little cooling aloft are frequently associated ith significant changes in the height of the subsidence inversion, or a disruption of it. he disturbances are rather similar to the polar ' trough ' described by Riehl (1945) but ere is less evidence that they are cold or occluded fronts.

During the summer differential heating between sea and land exerts a strong influence n the behaviour of ' cool changes ' which regularly break the hot weather in southern ustralia. Two, or sometimes more, discontinuities (change-lines) in sequence may company the passage of a main trough, the leading change being as a rule the more onounced. It has been shown (Berson, Reid and Troup 1957) that over a wide coastal gion there is a marked afternoon maximum in the frequency of arrival of single d leading changes and that this is consistent with a diurnal variation in the speed of ovement deduced from synoptic analyses. The variation in speed, which also has an ternoon maximum, could be explained as an effect of the diurnal variation in the hori-ntal pressure field produced by differential heating of coast and hinterland. It was ncluded that a sea-breeze effect is instrumental in activating and carrying on to land

a mass of cold air presumably separated from off-shore streaming continental air by a initially weak convergence line.* An outbreak of the main cold mass of high-latitud origin would thus have a front-like surge of modified continental air ahead of it.

To prove the correctness of the hypothesis of a coastal front is difficult, especiall since the necessary observations over the ocean are not available. There is, on the oth hand, much evidence for the occurrence in south-eastern Australia of pressure-jum lines of the Freeman-Tepper type ahead of cold fronts, the cooling with the leadin change being on occasions due to rainfall or evaporating rain, but more frequently to th incursion of shallow cold air overlying the sea. Evidence for non-advective wind surg affecting the hinterland as far as 200 mi from the coast and originating on, or related t sea-breeze fronts has also been presented by Clarke (1956) and Reid (1957).

In the present paper an analysis is given of measurements of undercutting cold a made for the purpose of studying in detail structural features of the surge front and th strength of the underflow. Although the data are of necessity biased towards condition prevailing during the summer in southern Australia, the results are thought to be mo generally valid.

2. The observations

The data were obtained in the course of field work carried out as part of a 'Coo change study.' During the summers 1953-54 and 1954-55 serial pilot-balloon ascen were made after passages of changes at Aspendale on the eastern shore of Port Philli Bay some 15 mi to the southwest of the Dandenong ranges. Aspendale is also at th southern end of a line of stations with self-recording instruments maintained by th Commonwealth Bureau of Meteorology and other organizations.

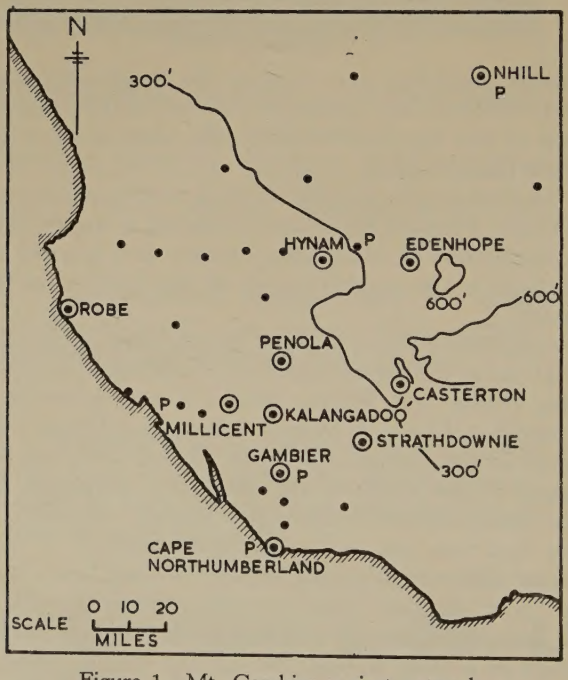


Figure 1. Mt. Gambier project network.

⊙ Major stations: equipped with at least wind-direction recorder, thermograph, hygrograph and barograph
• Minor stations: equipped with wind-direction recorder and thermograph.
P Pilot balloon ascents made.

* Formation and persistence of such coastal fronts over land, but in conditions rather different from those preceding the co changes, had been suggested by Cassidy (1945). According to Forder (1945), a frontogenic field of the col between migratin anticyclones over the sea might be responsible for the leading change, since the region of strongest thermal contrast extends fro the Northern Great Bight along the coast of South Australia, western Victoria and western Tasmania.

There were certain noteworthy recurring features of the sloping layer of concentrated nd shear and in the distribution of the velocity of the cold air, but they might have been al peculiarities. Subsequently a close network of stations with recording instruments s set up in a suitably flat region in South Australia and south-west Victoria bounded the south and west by comparatively straight coast lines (see Fig. 1). Synchronized ot-balloon observations at half-hourly to hourly intervals were made from a number of aces mainly in the rear of change-lines. Two-theodolite ascents were made at Penola d tail ascents at the other stations (see Fig. 1). Vertical motions could be deduced from e departure of the ascent rates from the theoretical or average measured values.

The data used in this paper relate to four cool changes, some of them ' double,' aversing large parts of south-eastern Australia. On three changes in 1954 ascents were ade at Aspendale, and on one change in January 1956 they were made from three places the South Australian project network.

In the five weeks during which this network was functioning, sea-breezes occurred equently. On one occasion a marked front formed a few miles inland. The data from is case are also used.

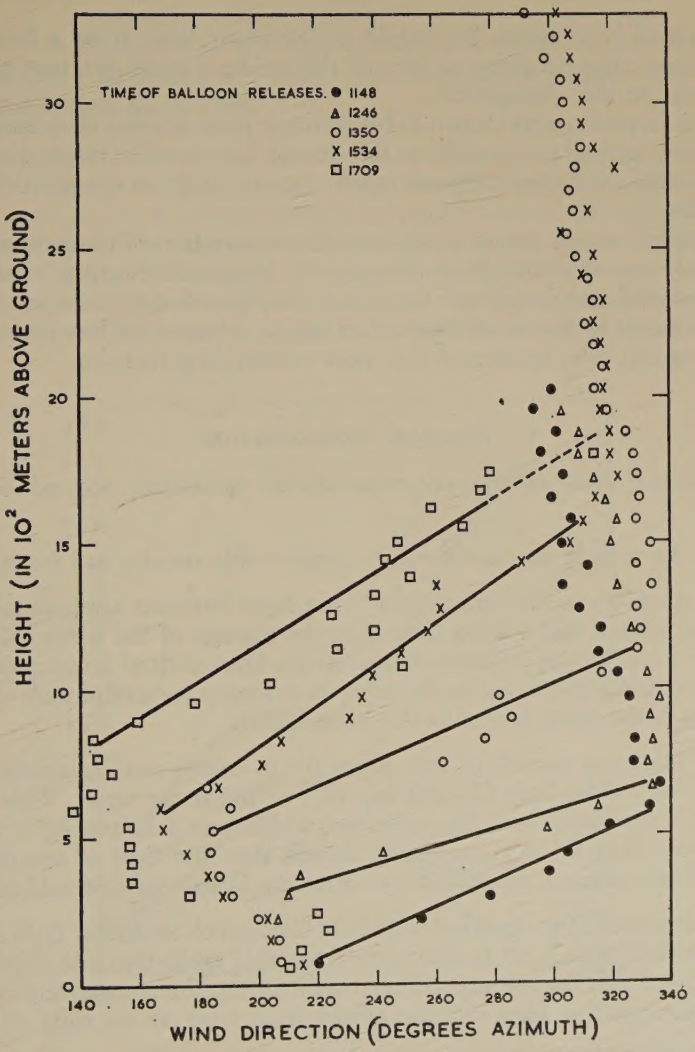


Figure 2. Determining the top of the shear layer from the wind direction Cool change at Aspendale 31 Dec. 1954, 1135 EST.

3. Determination of the elevated shear layer

It is a usual procedure to deduce lateral shear and convergence in the transitio zone near the ground from the horizontal velocities, because the transition layer is quas vertical close to the ground. Rapid changes of wind direction with height and associate wind shear arise from surface drag and internal friction, but are confined to a shallo layer well below the base of the elevated sloping shear layer except in the first few mile immediately behind the surface transition zone. Since beyond this region the slope c the elevated shear layer is of the order 1/100, the gradient normal to it of any propert may therefore be approximated by the vertical component. That layer is then for a practical purposes defined as the range of height through which the vector wind change most rapidly in the vertical.

One method of determining the base and top of the shear layer is to plot observe wind directions against height and draw a smooth curve through the points. This metho gives a fairly well-defined level at which the direction becomes quasi-constant with heigh and this level is taken as the top of the layer. An example is given in Fig. 2. (Incidentally in this example the ascents were more widely spaced in time than in any of the othe serial ascents.)

Another method is to assess the height of the shear layer from a hodograph plot The base appears on most occasions to be well defined by a rapid decrease of the velocity component normal to the change-line.

For each case isopleths were drawn in height-time plots of wind direction, wind speed and the components normal and parallel to the change-line (u and v respectively) as well as of the relative normal component u^*, where $u^* = u - c$ and c is the speed of movement of the change-line.

In the Aspendale series, errors arose from the assumption of constant ascent rate of the balloon. Such errors would affect the inferred height distribution of direction and velocity and so, possibly, characteristic features of configuration of the shear layer. However, the measurements on the South Australian region, which were free from the assumption of constant ascent rate, confirmed the more outstanding features.

4. Average configuration

The time-height plots of the above-mentioned quantities showed the following features :

(a) u exceeds c by a large amount through considerable depths and for several hours.

(b) There is a pronounced maximum of u^* in a layer between approximately 300 and 600 ft above ground and a short time after the passage of the surface discontinuity, followed by a continuous backing of wind in the layer with u^* subsequently decreasing and a strong maximum appearing in v. A tendency for a minimum of wind speed near the top of the shear layer has also been found.

(c) In none of the cases treated in this paper (or in others not included) was there a continuous rise of the base (Z) and top (Z') of the shear layer. This layer either levelled off temporarily or a dip occurred which was followed by a steeper rise. This rise was observed to occur about or just after the time of maximum u^* and therefore of maximum u, in other words after the initial burst of cold air.†

The phenomenon of a dip occurred with cold changes of dissimilar type and intensity as will be seen from Table 1. It is also clear from this table that the time lapse after passage at the ground (t_d), the corresponding distance from the change-line at the ground ($x_d = ct_d$) and the heights of base and top of the shear layer at the time of the dip, Z_d

† For convenience the transient interruption in the upward sloping of the shear layer will be referred to as the ' dip,' and the preceding portion as the ' nose.' Thus when a dip did not actually occur, the inflexion points in curves Z and Z' are taken to locate the singularity.

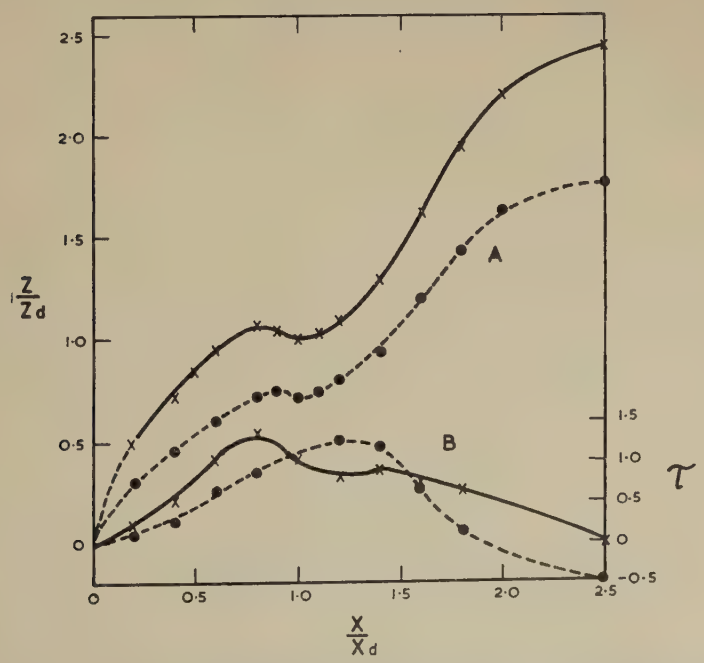
TABLE 1. SURFACE CHANGES AND THE 'DIP' IN THE WIND SHEAR LAYER

te	Place	Type of change	Time of passage (EST)	Wind shift	Temp. fall (°F)	Speed (m/sec)	Shear layer Z_d (decameter)	Shear layer Z_d' (decameter)	Shear layer t_d (hr:min)	Shear layer x_d (km)
				Cool changes						
54	Aspendale	Single	1216	W–SSE	8	6·0	24	37	1 : 15	27
54	Aspendale	Multiple	0825	NNW–WNW	2	3·5	52	74	2 : 55	37
			0900	NNW–S	5					
			1545	NW–S	4					
2.54	Aspendale	Single	1135	NNW–SW	16†	3·0	56	64	1 : 20	14
56	Mt. Gambier	Double	1545	NW–W	3	7·1	55	76	1 : 25	40
			1628	Nil	8	6·2				
56	Penola	Double	1538	NW–W	2	7·0	50	115	1 : 52	39
			1706	W–WSW	6	5·6				
56	Millicent	Double	1338	N–NW	0	6·8	102	116	2 : 20	59
			1600	WNW–WSW	12	6·8				
				Sea breezes						
56	C. Northumberland	Coastal	1255	Nil	1	4·5	18	–	2 : 25	21
56	Mt. Gambier	Inland	1500	NW–SSW	7‡	2·0	24	45	1 : 00	13
56	Penola	Inland	1722	NW–SW	11	2·4	27	–	1 : 35	33

† Pressure jumps observed *with* passage at Mt. Gambier, 1·7 mb; Essendon, 1·9 mb; East Sale, 0·7 mb.

‡ Pressure jump observed *after* passage at 16·45.

d Z_d', do not follow a unique pattern. It can however be shown that the position of e dip is not random, for a distinct pattern emerges in a non-dimensional representation the average shape. The average configuration from the four cool changes listed in ble 1 is shown in Fig. 3, curves A. These are curves for the base and top of the shear er and are obtained by taking the individual Z_d and t_d as reference parameters, ulating the ratios Z/Z_d along x/x_d and subsequently averaging them.



ure 3. Curve A – The dip in the elevated shear layer. Curve B – Distribution of relative horizontal mass transport (τ).

A measure of the scatter of the individual ratios Z/Z_d in their dependence on t was obtained by treating heights of base and top independently. The scatter was fou to be quite small, in the range $0{\cdot}6 < x/x_d < 1{\cdot}2$.

In the bottom part of Fig. 3 dimensionless curves are shown for the relative ma transport, in unit time and unit width, in the direction normal to the change line a directed towards it. This transport or ' inflow ' is defined as the integral of ρu^* extend from ground level to the base or top of the shear layer respectively. Since the decrease density (ρ) in the height ranges considered could be disregarded for the purpose of th study, the appropriate dimensionless quantity was approximated by the ratio $\bar{u}^* Z/\bar{u}_d^* Z_d =$ say. Here the bar indicates averaging from ground to the base or top of the shear lay respectively. In the distribution of τ the main points mentioned under (b) above may verified.

5. The friction head

The shape of the forward part of the shear layer resembles the friction head whi has been observed in experiments with stratified fluid in inclined open channels (Ippe and Harlemann 1952). In these experiments the front of the denser underflow releas into the less dense fluid has been observed to assume the shape of an elevated nose. is of course possible that in the present observations we deal essentially with a non steady phenomenon such as, for example, roll waves on an interface. If this were so, th actual shape of the shear layer could not be deduced from observations at one plac since distortions would occur in course of time and the space distribution could not b inferred from the local time changes. Evidence against such distortions will be given i Section 7. In this section it will be postulated that any distortions of this kind were ver small or very slow. The discussion is therefore concerned with constant-shape phenomena

Kármán (1940) deduced the form in which a mass of cold air breaks into a mass of warm air which might be assumed at rest. A description of such undercutting has been give by Prandtl (1952) and Fig. 4 is reproduced from Prandtl's book. In Fig. 4 (a) condition are simplified in that surface drag with its attendent feature of overhanging cold air clos to the ground is disregarded. Such overhanging is shown in Fig. 4 (b) where the stream lines, unlike those in Fig. 4 (a), are in a fixed system of coordinates : the relative stream lines of the large-scale motion would obviously not pass through the interface at an point and closed circuits can only occur above and below it.

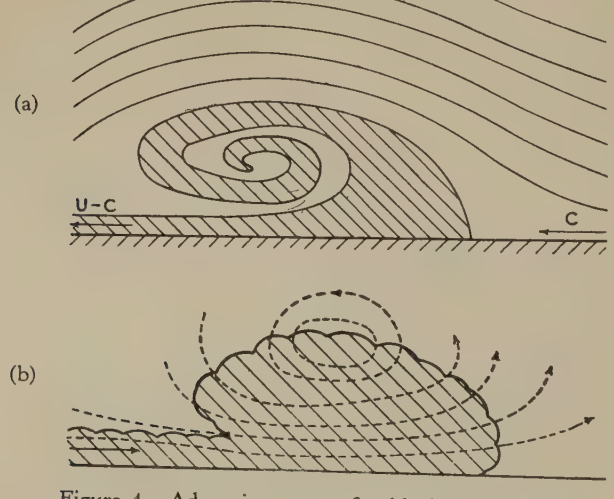


Figure 4. Advancing mass of cold air (after Prandtl).

(a) Flow relative to the front (idealized); (b) Flow relative to the ground, actual phenomenon.

Prandtl deduced the result that if the drag on the ground were disregarded, the tion were uniform within the cold air and the warm air initially at rest, the ratio c/u $- u^*/u$ is little different from $\frac{1}{2}$ and is equal to $\frac{1}{2}$ when the (potential) density difference ween the upper and lower mass is so small that the square root of the density ratio ght be put equal to unity. The latter condition is satisfied in practically all atmospheric d-front phenomena except when snow becomes mixed with the air as, for example, in avalanche over a precipice. In Prandtl's formula the speed u must of course not be en as that immediately behind the front.

It might be expected that the formula $c = u/2$ would be applicable to cold-air ges (fronts) over comparatively flat and uncorrugated ground such as sea-breeze nts in certain regions and the dry cool changes studied in this paper. Its verification m observational data could then be taken as an indication that the peculiarities in the m of the cold-air body, as deduced from time variations at a fixed place, would be re or less unchanged in time, since the formula is a direct consequence of Bernoulli's orem. In applying it to occurrences when the warm air was initially *not* at rest but in iform motion, u is to be replaced by $u - u'$, where u' is the normal component in the rm air. Since in reality the motion is varying with time and height in either mass, erages over time and height, $\bar{u}_m$ and $\bar{u}_m'$ must be substituted.

Serial pilot-balloon ascents were made by Koschmieder and collaborators oschmieder 1936) in a detailed investigation of sea-breeze fronts at the southern shore the Bay of Gdansk. These fronts moved predominantly *against* a generally off-shore adient wind. For a large number of cases Koschmieder gives the ratio c/u_0 where u_0 the normal surface-wind component behind the front, but in the wake of the initial st. It was possible to compute from Koschmieder's reconstructed space cross-sections the normal flow the relevant components $\bar{u}_m$ below the base in 4 cases. The following erage ratios c/u are obtained from the Gdansk sea-breeze data and the cool-change ta used in the diagrams of Fig. 3.

Sea-breeze fronts at Bay of Gdansk	28 cases	c/u_0	$= 0{\cdot}62$
	4 cases	$c/(\bar{u}_m - u_0')$	$= 0{\cdot}55$
Cool changes at Port Phillip Bay and in the Mount Gambier region	4 cases	$c/\bar{u}_m$ (to base)	$= 0{\cdot}50$
		$c/\bar{u}_m$ (to top)	$= 0{\cdot}58$

Inclusion of $\bar{u}'$ in the denominator of the last 3 quotients would reduce their values ightly, since in general $u' < 0$ (off-shore gradient warm air flow) and $|u| > |u'|$. The opropriate ratio appears therefore to be on the average close to 0·5, the value predicted y Prandtl's formula.

Referring back to the friction head, Prandtl makes no mention of its typical *dimension*. pen and Harleman (1952) communicated results obtained by Braucher (1950) experi- ientally by taking motion pictures of stratified flow in an open channel with a slightly nclined bottom. The front of the heavier underflow released into the lighter fluid in these xperiments, made with varying inclinations and density differences, was observed in ach experiment to move with essentially constant velocity and unchanged shape.

The main features of interest are the lifting of the nose of the surge front above the hannel bottom (overhanging), the relatively short length of the nose, and a rather abrupt hange to the lesser depth in the uniform flow behind it.

The physical explanation of the friction head is briefly as follows : while the lighter luid is being displaced upward a force must be provided to accelerate it against the esistance at the channel boundary and against the friction at the interface. As the initial orce will be larger than the gravity forces maintaining the subsequent uniform flow, this ecessitates an increase in the initial depth of the underflow. In the *atmosphere* too, round friction will cause a slight elevation of the nose, but the turbulent friction in the levated shear layer will be reduced because of the high static stability usually associated vith it. From this we may expect that the friction head in the atmosphere will be *omparatively* smaller than in the hydraulic case.

If the configuration of the atmospheric shear layer, observed as it was with a fai wide range of synoptic conditions, were the counterpart to a friction head in the laborat experiments on fluids, one ought to be able to describe the *basic* flow in both cases by c common dynamic parameter having about the same value. The appropriate parame is the modified Froude number, viz., $F = V\left(\frac{\Delta\rho}{\rho} g\, d\right)^{-\frac{1}{2}}$ for the stratified fluid, a $F_1 = U\left(\frac{\Delta\theta}{\theta} g\, Z\right)^{-\frac{1}{2}}$ for the atmosphere, V being the mean velocity and d the depth the uniform underflow which, it should be added, in the experiments was small compar with the depth of the upper fluid initially at rest; ρ and θ are the mean density and potent temperature respectively, $\Delta\rho$ the jump at the interface and $\Delta\theta$ the increase through t shear layer. The latter was estimated from the surface discontinuity of temperature a may be somewhat in error, whereas the values for θ are to better than 1 per cent. F w computed from tabulated values of V, h, ρ, and $\Delta\rho$ given by Ippen and Harleman.

TABLE 2. FROUDE NUMBERS IN STRATIFIED FLOW IN OPEN CHANNELS AND IN THE ATMOSPHERE

Channel flow			Atmosphere (Averages taken over the range $0{\cdot}6 \leqslant t/t_d \leqslant 1{\cdot}4$)					
				$100\,U/Z$		F_1		
V/d	F	Date		Base	Top	Base	Top	
0·71	0·36	8·1·56	A	0·60	0·42	0·50	0·41	46
1·10	0·45							
1·19	0·88	4·3·54	B	0·90	0·38	0·76	0·43	50
1·48	0·46	31.12.54	C	1·37	0·84	0·59	0·39	22
2·08	0·69	9·2·54	D	2·29	0·68	0·93	0·36	62
Mean 1·31	0·57	–		1·29	0·58	0·70	0·40	

In the expression for F_1 the quantity $U = \bar{u}_m - \bar{u}_m'$. The space averaging of u would be suitably extended from the top of the shear layer to the tropopause, which would then be regarded as a constant lid. No data were generally available to compute $\bar{u}_m'$. It was therefore necessary to make the assumption that $\bar{u}_m'$, the space- and time-averaged normal component of the wind above the shear layer, is very nearly equal to c, the speed of movement of the front. With this assumption† $U = \bar{u}_m - c = \bar{u}_m{}^*$.

It should be added that in taking F_1 as the significant parameter effects of vertical shear in the warm air, i.e. the baroclinicity of the upper flow, are disregarded and the pressure forces acting on the cold air are assumed constant in time. These two assumptions are considered the least restrictive.

The results of these calculations are summarized in Table 2. F and F_1 are tabulated against increasing ratios of velocity and depth of the underflow, V/d and U/Z_{base} or U/Z_{top} respectively, the first and third of which cover very much the same relative range. The average numbers F and F_1 differ little from one another, having the values 0·57 and 0·55. It is noteworthy that $F_{1\,\text{top}}$ varies little from case to case, but the significance of this is not clear.

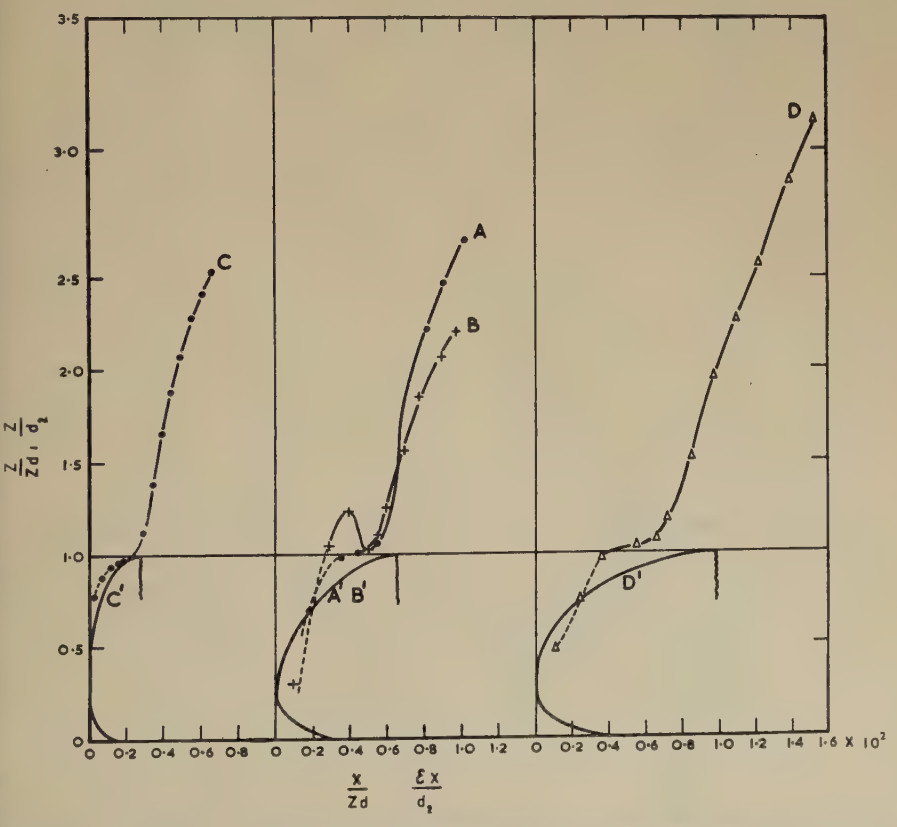
Having established the basic dynamic similarity of the flows, a comparison of the shape of the surge head is in order. In Fig. 5 the curves A, B, C and D are the configurations of the top of the shear layer associated with the cool changes listed in Table 1. They are dimensionless curves, the height Z_d being chosen as the single reference parameter. Ippen and Harleman found that the results of Braucher's experiments could be fitted to a single curve in the x/d_2, z/d_2 coordinate system. Clearly, this is not so with undercutting cold air.

† There is an analogue to this assumption in the theory of pressure-jump lines (Tepper 1952). In order to derive an analytical expression for the speed of a pressure jump in a two-layer barotropic atmosphere it is necessary to assume that the average (normal) velocity of the potentially warmer air aloft is equal to the speed of the pressure jump.

However, in view of the difference in order of magnitude of the horizontal dimensions ween the fluid experiments and the atmospheric flow – the difference in vertical ension is irrelevant in this respect since the depths d_2 and Z_d are the reference ntities – an appropriate change in the scale of the abscissa x/d_2 is required so that parison can be made. That change is made by dividing x/d_2 by Z_d/x_d. The relevant tiplying factors ϵ are given in the last column of Table 2. The curves A′, B′, C′ and D′ he lower part of Fig. 5 are reproductions of the friction head (Fig. 9 in the paper by en and Harleman) with the alteration in the scale of abscissa. For the curves A′ and B′ mmon factor 0·50 was used.

There is but a broad resemblance between the lower portions of the curves A to C the corresponding curves A′ to C′. The wind observations were clearly inadequate prove or disprove the occurrence of ' overhanging.' On the other hand, we note a king correspondence of the transition to *uniform* underflow with constant smaller th ($d < d_2$) to the transition to more or less *uniformly increasing* depth of the cold air.† e correspondence is seen in a consistent displacement of the points $z/Z_d = 1$ from the nts $z/d_2 = 1$ towards smaller x/Z_d, indicating shortening of the friction head in the osphere compared to that of the stratified fluid in the open channel. Earlier in this tion such differences were predicted from the effect of high static stability in the vated shear layer to reduce turbulent friction.

Beyond the friction head, Braucher's experiments indicated highly turbulent flow a region where the depth of the underflow decreases rapidly to the quasi-constant



gure 5. Friction head of an undercutting cold air mass (A, B, C, D) and of a heavy fluid in open channel flow (A′, B′, C′, D′, – abscissae adjusted)

† To the extent that in the region $z/Z_d > 1$ geostrophic equilibrium is predominant, the upward slope of the shear layer in t region is due to the effects of the earth's rotation and is essentially determined by Margules's formula.

value d. To date it has not been possible to show from the behaviour of the balloons similarly turbulent region in the vicinity of the dip or near the point $z/Z_d = 1$. Furth research employing cloud photography and radar techniques should be useful, as clou formation and precipitation patterns would be closely related to the friction head.

6. Characteristics of the inflow

The cumulative inflow into the 'wedge' of cold air is predominantly in excess the increase of mass due to the advance of the wedge. This follows from consideratio of the distribution of u^* and τ discussed in Section 4. In order that continuity of mass l preserved the inequality

$$\int_0^Z \int_0^t u \, dz \, dt > \int_0^Z \int_0^t c \, dz \, dt \quad . \quad . \quad . \quad ((1)$$

or $\Phi > M$, has to be associated with lateral divergence, distortion in time (c varyin significantly with height) or a circulation passing through the shear layer.

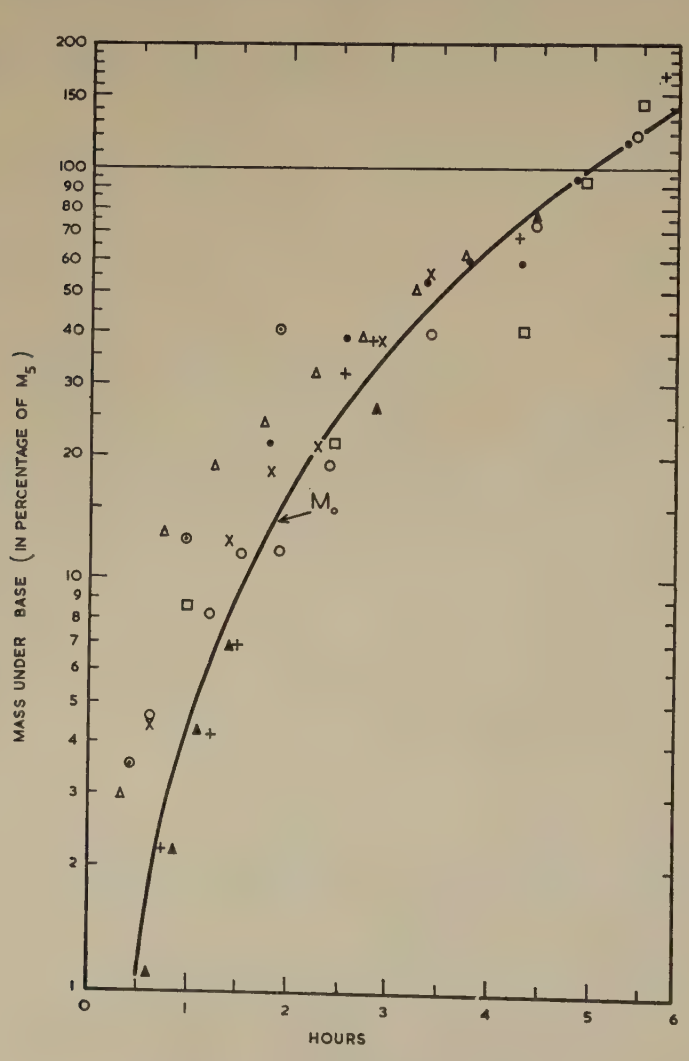
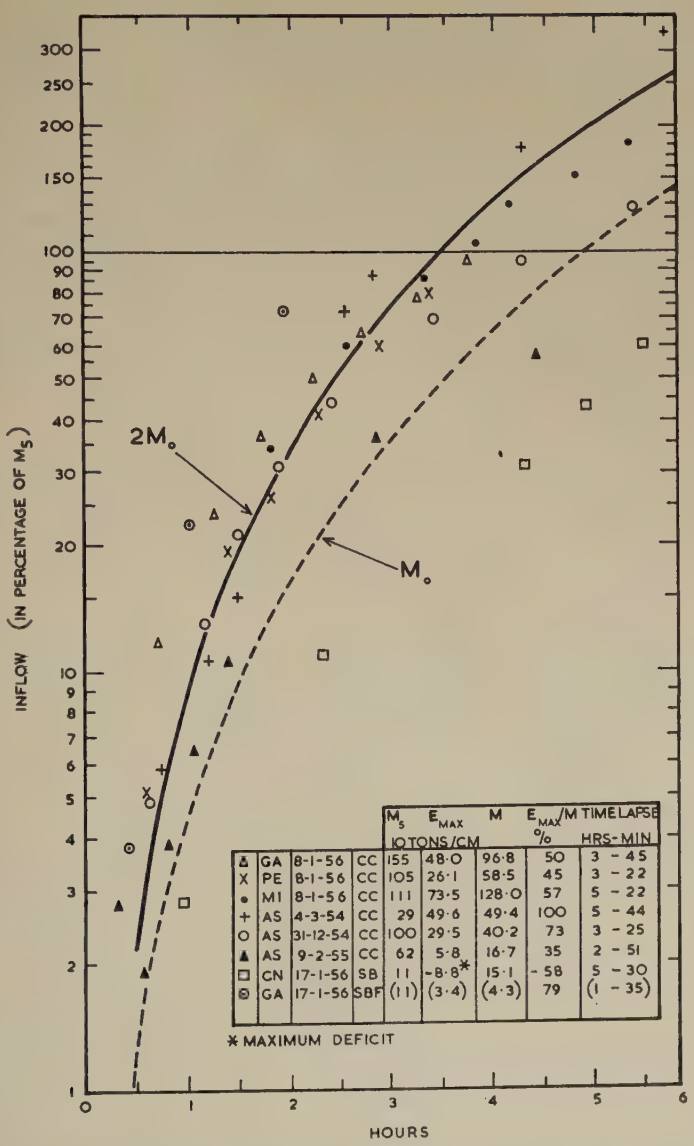


Figure 6. Increase of mass under shear layer.

Successive values of M and Φ, assuming c constant with height, were evaluated for the 9 ascent series listed in Table 1, omitting the last one at Penola. These quantities plotted in Figs. 6 and 7 respectively on a logarithmic scale as percentages of the mass obtaining at 5 hr after passage of the (leading) discontinuity. Some of the series did extend beyond 3 hr but in these cases M_5 could be assessed from an extrapolation of M curves. No appreciable error arises from this procedure since in plotting M against e smooth curves with a definite trend could be drawn, except in the immediate vicinity he dip in the shear layer which occurred after one to two hours (see Table 1).

For comparison the curve M_0 is drawn in the diagrams, M_0 representing the cumula- mass that would result if the height of the cold air (base of the shear layer) were to uniformly and in Fig. 7 the curve $2M_0$ is included as an indication of the departure n this simple model.



				M_5 (10 TONS/CM)	E_{MAX}	M	E_{MAX}/M %	TIME LAPSE HRS-MIN
Δ	GA	8-1-56	CC	155	48·0	96·8	50	3 - 45
X	PE	8-1-56	CC	105	26·1	58·5	45	3 - 22
•	MI	8-1-56	CC	111	73·5	128·0	57	5 - 22
+	AS	4-3-54	CC	29	49·6	49·4	100	5 - 44
O	AS	31-12-54	CC	100	29·5	40·2	73	3 - 25
▲	AS	9-2-55	CC	62	5·8	16·7	35	2 - 51
□	CN	17-1-56	SB	11	-8·8*	15·1	- 58	5 - 30
⊙	GA	17-1-56	SBF	(11)	(3·4)	(4·3)	79	(1 - 35)

* MAXIMUM DEFICIT

gure 7. Cumulative inflow under shear layer. In the inset table E_{max} is the largest computed excess inflow d M the mass under the shear layer at the time of maximum excess inflow, given in the last column as time lapse since the passage of the change.

The positions of points relative to the curve M_0 in Fig. 6 indicate by and large th well known convex forward edge of the under-cutting cold air mass. The friction hea can be perceived by following the points in each case; it is also seen in the general large positive departures of M from M_0 in the interval one to two hours approximatel

It is evident from the cluster of points along the curve $2M_0$ in Fig. 7 that large excea inflow E $(= \Phi - M)$ obtains after about two hours in most cases. However, the se breeze on 17 Jan. 1956 at the coastal station (Cape Northumberland) and the Februa cool change at Aspendale are associated with marked deficits of inflow. With the Decembo change initially positive E values decrease to almost zero at five hours after passag and progressive backing of wind under the shear layer, mentioned in Section 4 as a featu of average conditions, is particularly pronounced in this and in the December case. Th wind direction veers upwards through the shear layer by about 180° and the backing wit time constitutes the main contribution to the eventual deficit of inflow : the mass transpo into the advancing wedge is no longer sustained at the coast. These findings strongl suggest that a sea-breeze mechanism operated on a large scale. The February chang arrived at about noon, the surface wind backing from W to SSE (see Table 1). However the surface discontinuity penetrated inland well beyond the Victorian mountain range and thus it cannot be explained as a sea-breeze front in the usual sense but rather as bein caused by an activation of an off-shore coastal front. The frequent occurrence of such process had been inferred in the above-mentioned study (Berson, Reid and Troup 1957).

In the distinction between changes associated with incursion of cold air of distan origin and others of adjacent origin, the study of quantities such as the cumulative M values might be useful. It is seen from the inset table of Fig. 7 that there is a wide rang of M values. As would be expected from the much smaller depth of cold air as well a slower movements of sea-breeze fronts in general, by far the smallest value of M wa computed in the sea-breezes of 17 Jan. 1956.

7. Circulation and entrainment

In a field of horizontal convergence ascending motion under, and at the base of th shear layer is to be expected and a corresponding ' ana-circulation ' within the wedge o cold air is usually observed or deduced under a frontal inversion.

Thus Koschmieder (1936), in the studies of well-developed sea-breeze front mentioned above, concluded that a considerable portion of the sea-breeze air is entraine in the warm dry air aloft. The occurrence of inflow deficit with the sea-breeze of 17 Jan 1956 at the coastal station (Cape Northumberland) seemingly contradicts Koschmieder' results. However, a front did not develop until after the sea-breeze penetrated some distance inland where the off-shore gradient wind prevailed, and in fact marked cumulative excess inflow was apparent at Mt. Gambier aerodrome, some 20 mi inland (see Figs. 6 and 7, and the inset table).

The occurrence of mass transfer through an inversion layer has been demonstrated in the trade-wind regime (Riehl, Yeh, Malkus and La Seur 1951). The downstream rising of the inversion in a large-scale field of *divergence* and sinking of air columns was shown to be accompanied by a large-scale *downward* transfer of mass through the inversion. Moisture and heat are at the same time transferred upward by turbulent mixing and by penetration of cumulus convection through the base of the inversion.

Data for the study of mass transfer were available in the case of the double cool change of 8 Jan. 1956 which traversed the South Australian project network. As the leading change line simultaneously passed Mt. Gambier aerodrome and Penola and events at these places were much alike in several aspects, as will be seen from Table 1 and the inset table in Fig. 7, it was possible to use the triangle method for computing the relevant quantities from upper winds at Millicent, Penola and Mt. Gambier. Consider a fixed prism whose length (L), width and height are the distance from Millicent to the midpoint between Mt. Gambier and Penola, the distance between these two places and

base of the shear layer at Penola respectively. The horizontal divergence at discrete es was computed level for level in the prism and averaged vertically. The cumulative tical (upward) transport through unit width of the prism's lid A is then given from the ation of continuity, assuming constant density, by

$$W_A = -\bar{\rho} L \int_0^{Z_P} \int_0^t \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) dt\, dz \quad . \quad . \quad . \quad (2)$$

x co-ordinate pointing along L and the y co-ordinate to the right. Similarly, the nsport or *outflow* through the base (B) of the shear layer between MI and PE is given by

$$W_B = W_A + \bar{\rho} \int_{Z_P}^{Z_M} \int_0^t (u - c)\, dt\, dz - \bar{\rho} \int^{\Sigma} \int_0^t \frac{\partial v}{\partial y} dt\, d\sigma \quad . \quad . \quad (3)$$

ere Σ denotes the semi-prismatic domain cut out by A and B. As will be seen below last term on the r.h.s. may be disregarded as negligibly small, whence

$$W_B = W_A + \bar{\rho} \int_{Z_P}^{Z_M} \int_0^t (u - c)\, dt\, dz \equiv E_M - E_P - \bar{\rho} L \int_0^{Z_P} \int_0^t \frac{\partial v}{\partial y} dt\, dz \quad . \quad (4)$$

ere E is the cumulative excess inflow (see Section 6). Over the distance M to P the rage velocity at the base and normal to the shear layer in the time interval $(t_2 - t_1)$ is n proportional to the increments of W_B and given by $w_B = (W_{B,2} - W_{B,1})/t_2 - t_1)\,\bar{\rho}L$.

ABLE 3. HORIZONTAL CONVERGENCE (10^{-5} sec^{-1}) BENEATH, CUMULATIVE TRANSPORT (10 tons cm^{-1}) THROUGH AND VERTICAL VELOCITIES AT BASE OF SHEAR LAYER (cm sec^{-1})

Time lapse	Horizontal convergence (Prism A)			Cumulative transports			Vertical velocities at base of shear layer			
r : min)	$-\partial\bar{u}/\partial x$	$-\partial\bar{v}/\partial y$	$-\mathrm{div}_2 V$	W_A	W_B	w_B	Average†	GA	PE	MI
						8				
: 24	1·4	− 2·4	− 1·2	0·5	10·4		14	10	40	5
						20				
1 : 51	0·4	− 1·5	− 1·1	− 0·1	26·5		− 24	− 20	–	− 35
						− 10				
2 : 22	− 0·4	1·8	1·4	− 0·1	17·7		− 29	− 15	− 60	− 20
						5				
2 : 52	2·3	5·4	7·7	4·5	22·1		7	–	5	10
						2				
3 : 22	1·4	4·9	6·3	10·8	23·5		21	45	25	10
	Average over 3 hr 22 min					4	8	9	19	1

† Weighted average = $\frac{1}{2} w_{MI} + \frac{1}{4}(w_{GA} + w_{PE})$.

The results of these calculations are summarized in Table 3. The values in the first ree columns must be taken as giving merely an indication of their order of magnitude. nce this order conforms to the horizontal scale of motion considered ($L \sim 50$ km) ere is some assurance that the computed W_A are not much in error. The second term the r.h.s. of Eq. (3) is independent of the spatial distribution of wind and rather small rors are involved in its evaluation. It is seen that up to about three hours after passage W_B exceeds W_A as well as its components, by an order of magnitude. Since $\partial v/\partial y$ does ot increase by an order of magnitude between A and B, the third term on the r.h.s. of q. (3) could be disregarded as negligibly small.

It is seen from Table 3 that the average component w_B over three and a half hour is 4 cm sec^{-1}. The average vertical velocity deduced from ascent rates (see section 4) 8 cm/sec. Now $w_B = w \cos \alpha + (u - c) \sin \alpha$ where w is the vertical velocity and α th slope angle of the shear layer. From the average u at the base and average slopes, w_B found by this method to be 12 cm sec^{-1}. Therefore both this method and the one embodie in Eq. (4) give a significant motion normal to the base and directed upward. In view c the limitations in the accuracy of the observations and especially of the deduced vertica velocities, the discrepancy between the computed w_B might be considered as tolerable.

It is also seen from Table 3 that there are simlar alternations between positive an negative values of w_B and w which in view of the relative constancy of the quantit $(u - c) \sin \alpha$ is of some significance.

The discrepancy between the individual vertical *displacements* ($|w|$) could well b accounted for by a tendency of the balloon to be caught in convective updraughts (thermals and downdraughts superimposed on the large-scale vertical motion. On the other hand penetration of updraughts into the shear layer was not sufficiently widespread and intens to invalidate the inferences on large-scale mass transfer through the shear layer. Fo there was practically no cloud *below* the layer; 1/8 to 3/8 cumulus based at about 9,000 f occurred at some stage above it, apart from some medium and high clouds.

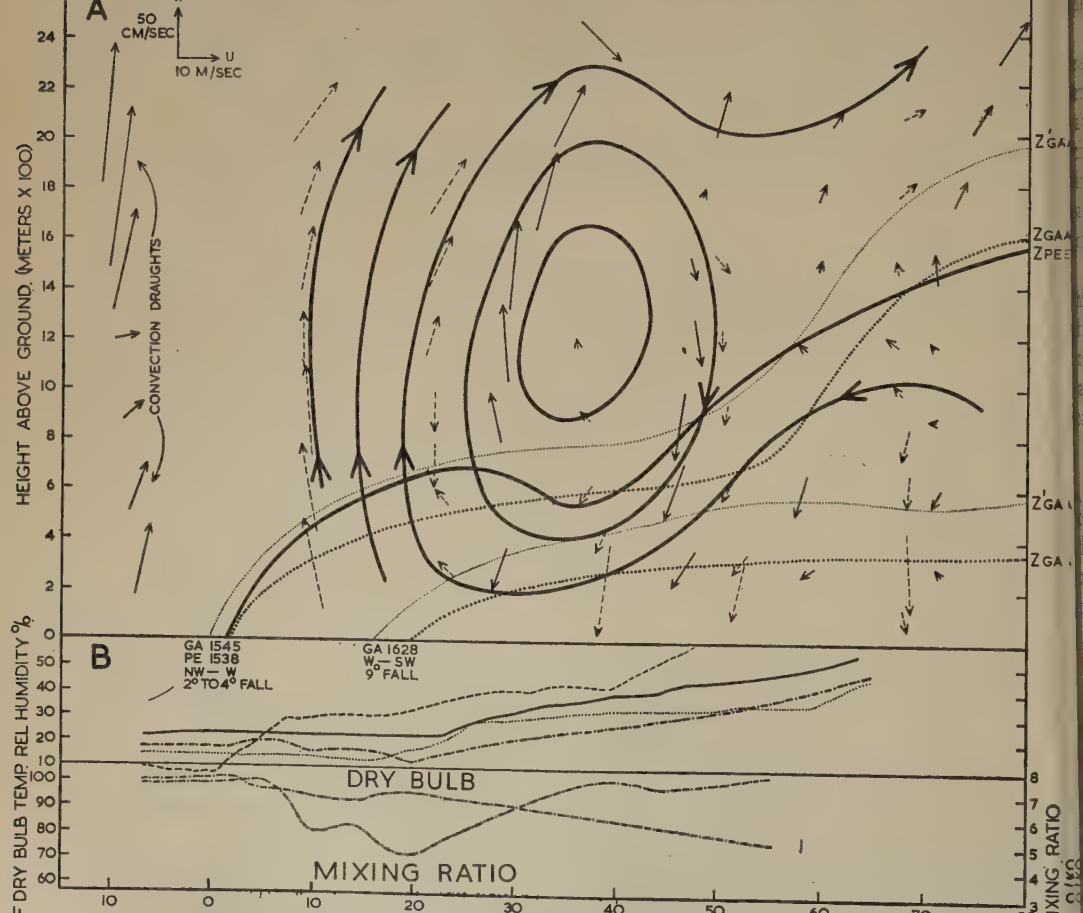


Figure 8. Converted time cross-section through the double cool change of 8 Jan. 1956. (a) Relative flow in normal vertical plane; (b) Changes at ground level.

The predominantly ascending motion at the base of the shear layer cannot be readily rpreted as a distortion *in situ*, since it would imply an enormous bulging of the shear r between *MI* and *PE*, the ratio W_B/M attaining the value 0·4 at about $3\frac{1}{2}$ hr after sage of the leading line at *PE*. Also, by that time the friction head had passed the on (see Table 1).

The evidence thus points to considerable non-convective mass transfer directed in e regions upwards, in others downwards through the shear layer and associated ersion, the upward transfer markedly outweighing the downward transfer. This l of entrainment is demonstrated more directly in Fig. 8 (upper portion) showing the tion in a framework moving with the speed of the change-line. The vectors $- c)\,\mathbf{i} + w\,\mathbf{k}$ are placed in their respective position. These vectors are drawn to the e of the coordinates so that $(u - c)/w = x/z = 1/25$. The lower shear layer, shown for Gambier, is directly associated with the second (trailing) change-line. (The existence uch a shear zone does not invalidate the main results of this and the preceding sections e the considerations are based not on momentum balance but on mass continuity).

The stream lines of the relative normal flow drawn roughly to fit the vectors indicate ending air through the forward part of the head, descending in the vicinity of the dip l sliding down along the upper boundary of the trailing shear layer, completing in this nner a *kata-circulation*. Further upstream in the region of the increased slope the tive motion is on the whole slower and again upward through the leading shear layer.

Supporting evidence for the kata-circulation is provided by the changes at ground el shown at the bottom of Fig. 8. With the passage of the leading wind-shift line the perature falls slightly, the relative humidity remaining unchanged or falling to lower ues than observed in the desiccated seaward flowing air of continental origin. The ing ratio decreases appreciably. The phenomenon is observed at all places not situated ectly on the coast. At Strathdownie, for example, the decrease of mixing ratio after the ssage of the leading change is almost 3 g/kg. At Cape Northumberland, situated on the st, the passage of the same line causes a sharp increase of mixing ratio by almost 4 g/kg.

The general decrease of mixing ratio in the maritime air as it travels some 30 to 40 mi and appears far too large to be accounted for by convective mixing between a very llow mass previously moistened by contact with the sea surface and the drier air eaming in the same general direction above it but below the shear layer. The decrease her indicates that the deduced circulation did not commence until after the cold air ssed on to land.

Conclusion

A series of observations of upper winds in the rear of surface discontinuities of the ld-front type has been described. These observations yielded detailed information on e shape of the forward portion of the undercutting mass and especially on the existence a friction head. They also revealed significant discrepancies between normal inflow to, and the change of mass under the elevated shear layer, but were insufficient for draw- g more definite conclusions on the effects of distortion, horizontal divergence beneath, d mass transfer across that layer.

The unique observational data in the case treated in the last section point to the st-mentioned factor as the most important. In this particular case the trailing discon- nuity might be regarded as the primary front but difficulties in this interpretation are e extreme shallowness of the cold air behind the trailing change and the maintenance the leading shear layer in the face of the overlying circulation.

Acknowledgments

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References

Berson, F. A., Reid, D. G. and Troup, A. J.	1957	*C.S.I.R.O. Div. Met. Phys. Tech. Pap.*, No. 8.
Braucher, E.	1950	' Initial characteristics of density current flow,' S.M. Thes M.I.T.
Cassidy, M.	1945	*Weather Div. Res. Bull.*, Melbourne, No. 1, p. 5.
Clarke, R. H.	1956	*Austr. Met. Mag.*, **11**, p. 77.
Forder, D. H.	1945	' Analysis and forecasting in the SW. Pacific Area,' Bure Met., Melbourne.
Gibbs, W. J.	1945	*R.A.A.F. Met. Serv., Tropic. Weath. Res. Bull.*, No. 13.
Gibbs, W. J., Gotley, A. V. and Martin, A. R.	1952	*Austr. Nat. Antarctic Res. Exp. Rep.* D, **1** (c), p. 67.
Ippen, A. T. and Harleman, R. F.	1952	Washington Semi-cent. Symp. on ' Gravity waves,' p. 78.
von Kármán, T.	1940	*Bull. Amer. Math. Soc.*, **46**, p. 615.
Koschmieder, H.	1936	*Forsch. Arb. Obs. Danzig*, **8**.
Palmer, C. E.	1942	*Prof. Notes, N.Z. Met. Off.*, No. 1.
Prandtl, L.	1952	*Fluid Dynamics*, London (Blackie), p. 368.
Reid, D. G.	1957	*Austr. Met. Mag.*, **16**, p. 23.
Riehl, H.	1945	*Univ. Chicago Misc. Pap.*, No. 17.
Riehl, H. Yeh, T. C., Malkus, J. S. and La Seur, N. E.	1951	*Quart. J. R. Met. Soc.*, **77**, p. 589.
Tepper, M.	1952	*U.S. Weath. Bur. Res. Pap.*, No. 35, p. 47.
Troup, A. J.	1956	*Austr. Met. Mag.*, **14**, p. 1.

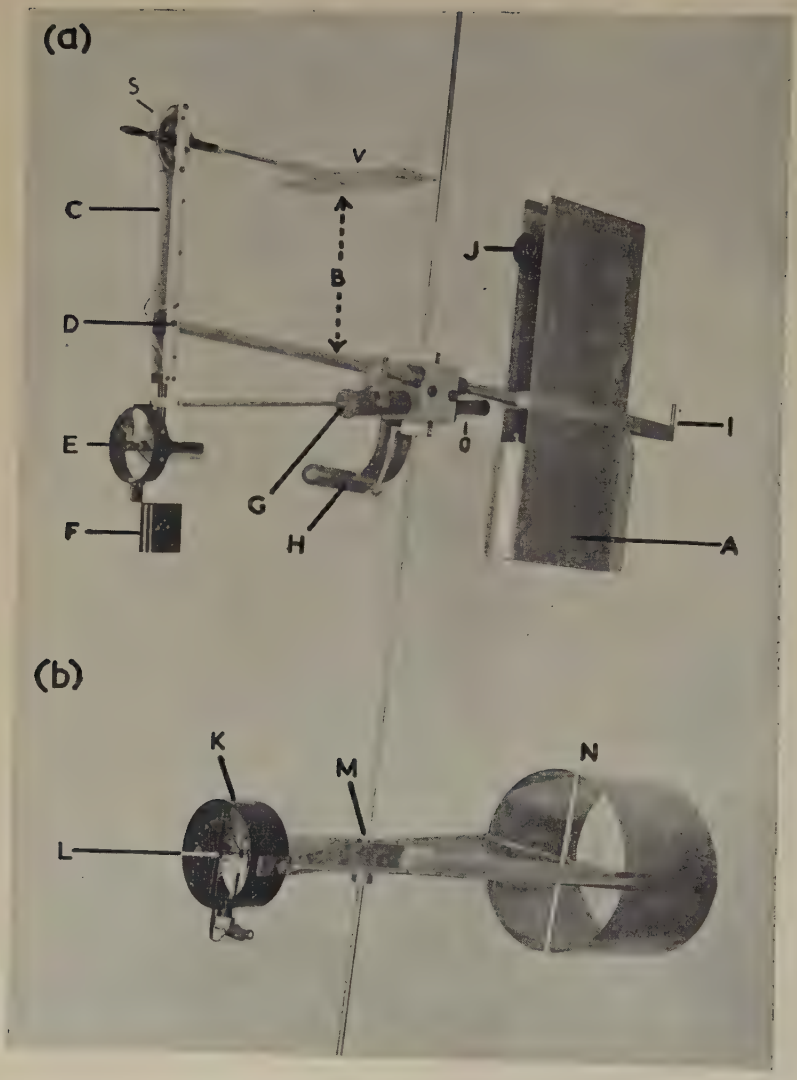

Figure 1. Instruments for measuring inclination of wind (a) and total speed (b), from a captive balloon cable.

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The measurement of gustiness in the first few thousand feet of the atmosphere

By J. I. P. JONES and H. E. BUTLER

Meteorology Section, Chemical Defence Experimental Establishment, Porton Down, Wilts.

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Summary

A description is given of instruments which have been designed and used to measure gustiness, in the n of inclination and total speed of the wind, from a position on the cable of a captive balloon. A light-ght wind vane with automatic compensation for tethering cable angle, and a sensitive vane airmeter used as sensing heads. The data, with reference signals, are relayed by electric cable to ground level recorded on multi-channel recorders. Both instruments have good response to wind fluctuations with ods down to one second.

A brief account is given of some observations made at Cardington, Beds., and some examples are pre-ed of records made at 1,000, 2,000 and 3,000 ft in contrasting meteorological conditions.

1. Introduction

In connection with certain diffusion experiments a requirement arose for instruments measure gustiness at heights beyond the reach of masts or towers. It was therefore essary to design the instruments in a form suitable for suspension from a captive loon, and in order to use the near-maximum operational height of the balloon, the weight equipment together with any connecting electric cable had to be kept low compared h that of the tethering cable. The weight could have been reduced by using hot-wire truments but these are fragile, the output is normally non-linear, and the calibration quently unreliable. The use of radio for transmission of data was considered as a means reducing the loading of the balloon, but was rejected because of the complexity of the ipment.

The first emphasis was on the measurement of the vertical component of gustiness d this requirement was met by the use of a light-weight vane responding to variations wind inclination, its angular position being registered electrically. To eliminate the ect of the changing angle of the tethering cable, the instrument was effectively sus-nded, a principle previously used by Field and Warden (1933). The instrument also corporated a vane anemometer which provided reference pulses for the setting of sen-ivity and from which mean wind speed could be determined. For many purposes it ems likely that an acceptable approximation to the vertical component would be provided the wind inclination and mean speed. However, for accurate values of this component was essential also to measure the fluctuation of total or horizontal speed, and for this rpose a sensitive Ower airmeter was modified.

A number of observations have been made using the captive balloon facilities at rdington, Beds. It is not the purpose of the present paper to give any detailed dis-ssion of the results, especially as much of the analysis has yet to be completed, but in ew of the novel nature of the observations it was thought appropriate also to give some amples of the raw data.

2. Instrumental details

(a) *Fluctuations of wind inclination*

The inclination vane, V, (Fig. 1 (a)) was mounted to respond to vertical wind fluctua-ns and positioned the sliding contact, S, of a low-friction potentiometer. Electrical nnection to the sliding contact was made by a hair spring. The potentiometer formed o arms of a bridge network which gave an output voltage directly proportional to the

vane inclination. The vane, and anemometer, E, were mounted at either end of an arm, which was maintained in the cross-wind vertical plane by suspension at D from tt arm of the control vane, A. Freedom of the control arm to rotate axially through 4 ensured that the mounting of the sensing heads remained in the along-wind verti plane when the balloon cable moved out of this plane. Damping of any tendency oscillatory motion of the arm, C, was accomplished in the along-wind plane by a balanc air piston-cylinder, G, and in the cross-wind plane by an oil damper, O. The assemb was mounted on the balloon cable by means of split bearings and was free to turn into t mean wind direction under the action of the control vane, which also provided accommod tion for a 3-volt battery and various circuit components.

The inclination vane, of aerofoil section (15·2 cm × 7·6 cm), was constructed balsa wood and tissue paper, doped and waterproofed, and with the 15·2 cm long dow support weighed 4·5 g. Wind-tunnel tests were made to select the minimum clearanc B, for the inclination vane which gave free exposure with inclinations within the ran + 40° to − 30°, and as a result the support, C, for the potentiometer was made 20 c long from its fulcrum, D. The anemometer, E, at the lower end of this support was standard pattern (by Negretti and Zambra), but was modified to include a step-ca operated switch which alternated the output voltage of the potentiometer with a referen voltage taken from a tapped resistor across the bridge. This reference voltage w equivalent to an inclination of − 20°. The cam was driven by the anemometer rot through fine gears so that the switch was operated at approximately every 300 m of a

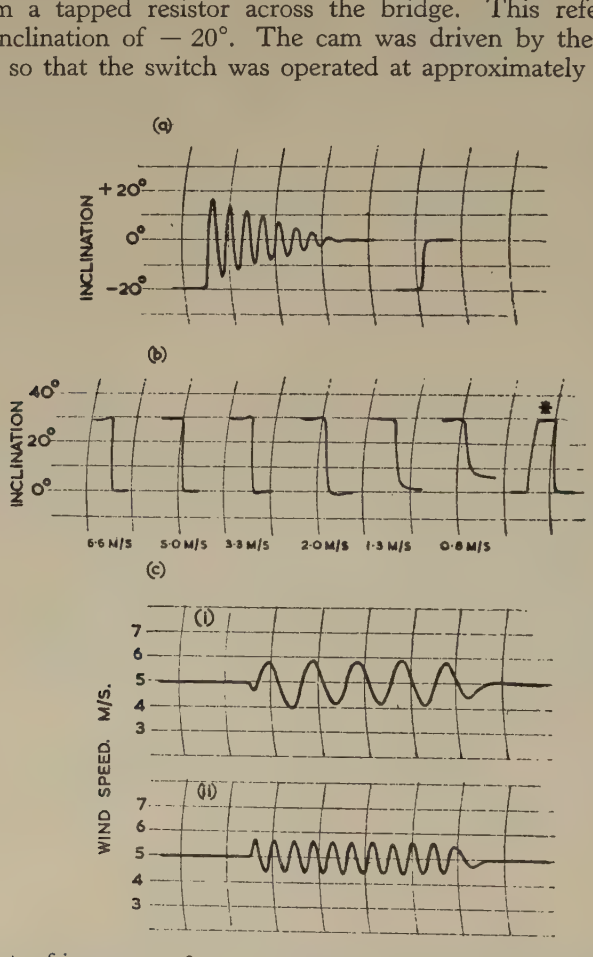


Figure 2. Response tests of instruments for measuring inclination of wind and total speed (the time-scal for all cases is 2·5 sec for one division). (a) Action of piston in damping oscillatory motion of potentiomete support. (b) Response of inclination vane at various wind speeds, and response of recorder only (*) to 1·25 sec pulse. (c) Airmeter response to approximately sinusoidal fluctuations of 2 m/sec peak-to-pea amplitude, and period : (i) 2·4 sec, (ii) 1·0 sec.

w for approximately 4 m of air flow. The output from the bridge circuit thus indicated e wind inclination for 99 per cent of the time and for the remainder gave pulses which libratedfor wind inclination and enabled the wind speed to be determined. The weight the anemometer and the additional streamlined weight, F, maintained the support arm rtical in wind speeds up to 13 m/sec. The natural period of the pendulum was 0·9 sec, hile critical damping by piston, G, prevented oscillatory motion. Fig. 2 (a) shows the fect of initially deflecting the pendulum 20°, without and with the damper connected.

A counterbalance weight, H, restored loss of balance due to the small (2·2 cm) ff-setting of the main support arm from the balloon cable, and the addition of a hook, at the tail provided an initial support for the electric cable, and a fixing point for cables ading from instruments at a higher level.

The starting torque of the vane bearing and potentiometer was 4 g cm and frictional ffects, though adverse at very low wind speed, were not much in evidence at normal orking speeds. Response tests were carried out at various speeds in a tunnel, by displacing e vane with a fine thread, to 30° relative to the air stream, and releasing it instantaneously. ig. 2 (b) shows the nature of the record obtained for 6 wind speeds over a range 6·6 0·8 m/sec and, for comparison, the response of the recorder alone. Though the vane as not dead-beat above about 3 m/sec over-shoot was not registered, owing to the ower response of the recorder. Under usual working conditions (5 m/sec or greater), he effective time of response was thus governed by that of the recorder (0·2 sec), but at m/sec it increased to about 0·5 sec. The vertical gustiness meter was considered uitable for use in wind speeds within the range approximately 3 to 12 m/sec, the lower imit being governed by the inadequate response of the control fin, and the upper limit y vibrations of the balloon cable which occurred at times in high wind speeds.

In practice 3 instruments were used simultaneously at various heights and in order to conomize in electric cable, the balloon tethering cable was used as a common conductor. he remaining conductors, including one from a thermistor circuit for measuring mean ir temperature, were brought to a junction near the lowest instrument, and from there ormed the 4 conductors of a 5,000 ft length of light-weight cable. The outputs of the otentiometers were amplified using amplifiers designed to maintain linearity, and records ere made on a three-channel recorder with full-scale deflection from the centre zero or 2½ mA and coil resistance 600 ohms. The sensitivity was normally adjusted so that one mall division (0·7 mm) corresponded to 2 degrees of inclination. A switch at the input o the amplifiers allowed recordings to be made of either inclination of each of the 3 vanes r the difference in the inclination of pairs of vanes.

(*b*) *Fluctuations of total wind speed*

The function of the anemometer was firstly to provide reference pulses for the setting f sensitivity, and secondly to give wind speeds. The minimum period over which the ean speeds could be measured was inversely proportional to wind speed and at 5 m/sec was about 1 min. For shorter-period fluctuations, a separate airmeter (shown at K in Fig. 1 (b)) was used. This was an Ower (1949, p. 150) airmeter with the gear train and dial removed, and with a miniature switch, L, fitted on the spindle of the rotor. Four contacts per revolution of the rotor were made, and the electric pulses produced were ' counted ' electrically over 2½ sec sequential periods and recorded as amplitudes on a recorder similar to that used for wind inclination. Provision was also made for continuously recording the pulse rate so that periods of shorter duration could be examined.

The performance of the airmeter was investigated in a wind tunnel, with various angles of pitch of the vanes, and its time of response measured on a high-speed recorder. The pitch of 30° (relative to the airflow) which was finally chosen gave a 90 per cent response time from zero of 0·3 sec, and a rotational speed of approximately 12/sec in a wind speed of 5 m/sec. Linearity of the calibration curve remained unchanged over a calibrated range of 1 to 10 m/sec, and the pulse rate (four times the rotational speed) varied from about 10/sec at 1 m/sec to 100/sec at 10 m/sec.

Tests of the airmeter to a fluctuating speed were carried out in the tunnel by moving the airmeter on a guide rail, to and fro along the airstream, with approximately sinusoidal motion. This was controlled in one direction with a 6 m length of string attached to a rotating arm fitted to a constant-speed motor, and in the reverse direction by an elastic cord. Tests at two frequencies were made, by altering the gear ratio of the motor, and peak speeds of 1·0 m/sec were used in both cases by adjusting the effective length of the rotating arm. Tracings of the records produced are shown in Fig. 2 (c). In a mean wind speed of 5 m/sec a peak-to-peak response of 90 per cent was obtained with a period of 2·4 sec and just over 70 per cent when the period was 1·0 sec.

For operation on the balloon cable, the airmeter support was carried on double bearings M (Fig. 1 (b)) which allowed it to be freely directed into a mean wind under the action of a cylindrical fin N. The total wind speed was thus measured, corrections being unnecessary for large deviations of wind inclination and direction (see Ower 1949, p. 152 for the effect of yaw). The airmeter was connected to the ground equipment with a separate light-weight twin-core cable.

The electronic integrator, for giving mean values over intervals of $2\frac{1}{2}$ sec (see Fig. 3), consisted of a high-speed relay, a storage and switching unit and a d-c amplifier. The relay, operated by an 18 V power supply and the airmeter switch, charged one of three equal condensers C_1 from a constant voltage supply, E, by step-transfer of charge via a small condenser C_2. The ratio of C_1 to C_2 was large (2000 : 1) so that the charging rate of C_1 was almost linear over a large range. Connections to the condensers C_1 were made through a 3-pole 'snail' cam switch S_1, driven continuously by a mains synchronous motor at 1 rev/$7\frac{1}{2}$ sec. Contacts 'a,' 'b' and 'c' of this switch dropped from segment to segment in sequence, but separated in time by only about 1/100 sec. In the position shown in Fig. 3 one of the 2 μF condensers C_1 was being charged through contact 'b,' another was registering its voltage on the recorder through contact 'c' and the d-c amplifier, while the third had been discharged through 'a.'

Every $2\frac{1}{2}$ sec 'a' dropped first, started discharge of the registering condenser and time-marked the recorder. Contact 'b' followed and transferred charging to the newly discharged condenser, and finally contact 'c' operated, registering the voltage which the condenser under charge had acquired during the previous $2\frac{1}{2}$ sec. A manually-operated switch S_2 allowed a desirable negative bias to be applied to the condensers C_1, and this enabled the full range of the recorder, which normally has a centre zero, to be used. For a

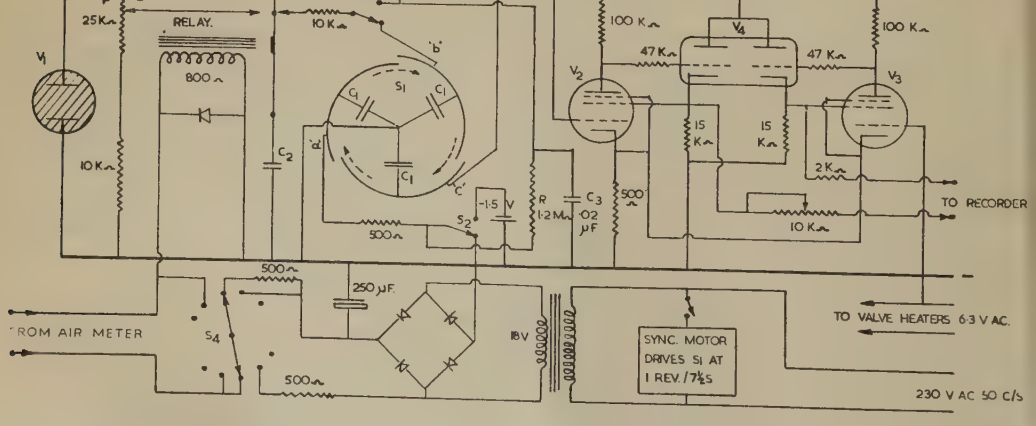


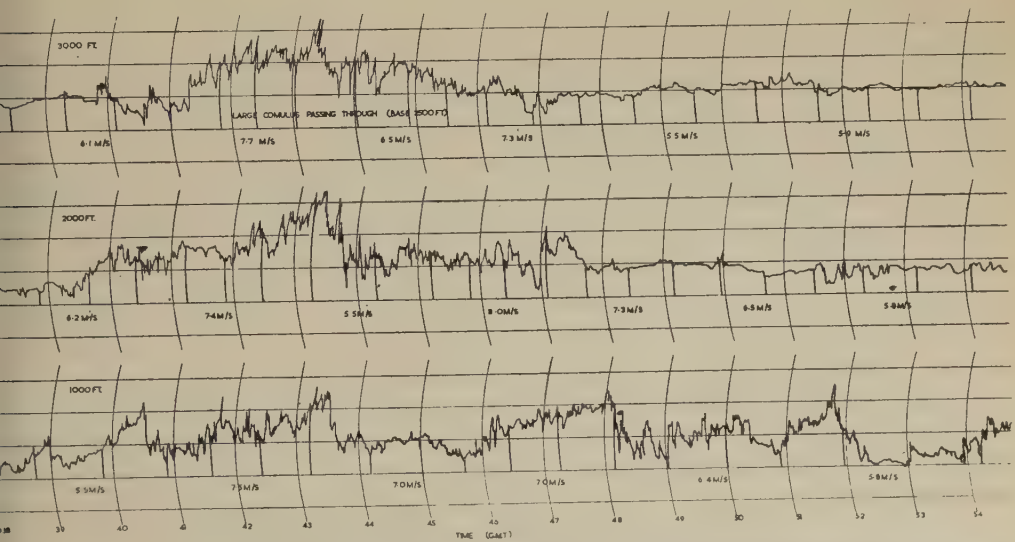
Figure 3. Circuit for counting airmeter revolutions over $2\frac{1}{2}$ sec sequential periods.
V_1 = 7475 (Stabilizer); V_2, V_3 = EF 37A; V_4 = 6S N7.
Relay = Siemens H73; C_1 = 2 μF; C_2 = 0·001 μF.

nuous registration of wind speed, within the limits of the airmeter response, conden- C_1 were replaced by a leaking condenser circuit, R C_3, by means of switch S_3. Setting- nd checking procedure included the operation of the relay at 50 c/s from a biased supply, with switch S_4. This allowed the short-term stability of the mains frequency e checked, and adjustment of sensitivity to be made by altering the input voltage, E, he integrator, with potentiometer P. In practice the sensitivity was usually adjusted hat one small division on the chart (0·7 mm) was equivalent to 0·2 m/sec.

(c) *Other observations*

A thermistor (Stantel type K 2361/120), mounted in a bakelite tube, J, (Fig. 1 (a)), fitted in the upper part of the control fin of one of the instruments, for the measurement ir temperature. To make effective use of natural ventilation a low operating current used, and the insulating mounting tube (3·2 cm internal diameter, 15 cm long) and ninium sides of the fin were utilised as radiation shields. At first the resistance of thermistor was measured with a Wheatstone Bridge, and, after deduction of the known e resistance, converted to temperature from a laboratory calibration of the thermistor. s method of observing temperature was later changed to a direct-reading method, h a bridge circuit mounted in the fin of the instrument, where the supply battery of gustiness instrument was used for power. Components were arranged so that the -of-balance current of the bridge had a near-linear relationship with temperature and e full-scale deflection on a 20-0-20 μA meter over a temperature range of + 30°F + 70°F. An absolute accuracy of $\pm \frac{1}{2}$°F was achieved without the application of cor- ions, and changes of 0·1°F could be measured. With this direct-reading method vas possible to make temperature readings with little delay and at frequent intervals ing the ascent, it being necessary to wait only about 1 min for the thermistor to acquire bient temperature.

A movement of the balloon, and hence of the instrument support, introduces errors the recorded values of gustiness. The effect of the changing angle of the tethering le due to this movement was eliminated from vertical gustiness measurements through principle of operation of the instrument, while for total speed fluctuations the operation he airmeter was independent of the cable angle. Errors caused by the bodily transfer the instruments through the air were, however, present in the recordings from both



ure 4. Sample record of wind inclination at 1,000, 2,000 and 3,000 ft at Cardington, 27 June 1956. Weather fine, with 4/8 Cu and Sc. base 2,500 ft.

instruments whenever significant movement of the balloon took place. To minimize effects of movements, and to enable approximate measurement of their value to be m easily, it was necessary to use a balloon which provided a substantial residual lift wher operating height. The tethering cable was thus maintained sensibly straight and culations of the height of the instruments could be made from a knowledge of the ca length and its angle, θ, from the vertical.

During experiments carried out at Cardington a record of movement was ma with the aid of a theodolite, with a potentiometer coupled to its telescope, erected n the tethering point of the cable. A point on the cable, usually one of the instrumer was ' followed ' continuously when recordings of gustiness were being made. For measu ment of height a cosine potentiometer was used and cos θ was recorded on a linear sc over a range 0·9 to 1·0. The full scale thus represented a height range of 10 per cent the distance, l, of an instrument from the winch (taken to be the length of cable), and close approximation to height changes was given from changes in $l \cos \theta$. After total spe measurements were introduced the cosine potentiometer was replaced by a linear o and the cable angle was recorded.

In conditions where there was little or no measurable vertical component of wi near the balloon, movement of the instruments was correspondingly small and in gene no corrections were necessary for accurate interpretations of the records. This present the best method of obtaining error-free measurements in turbulent air, i.e., by flyi the balloon where possible in relatively stable air above the turbulent region, with t instruments attached to the cable at selected heights above the ground.

3. Some operational details and examples of data

Observations have been carried out during a number of periods over the past ye usually with a ' Mk. 11 ' balloon which gave a lift of about 750 kg near the ground and residual lift at 5,000 ft of about 500 kg.

The gustiness meters were attached to the balloon cable usually at spacings of 1,000 for recording actual wind inclinations, and at spacings of up to 400 ft for recording t differences between the inclinations. Recordings were made at various heights up to maximum of 7,000 ft. The electric cable from a junction at the lowest gustiness met was 5,000 ft long, and was stored on a drum with its inner end accessible. Connecti

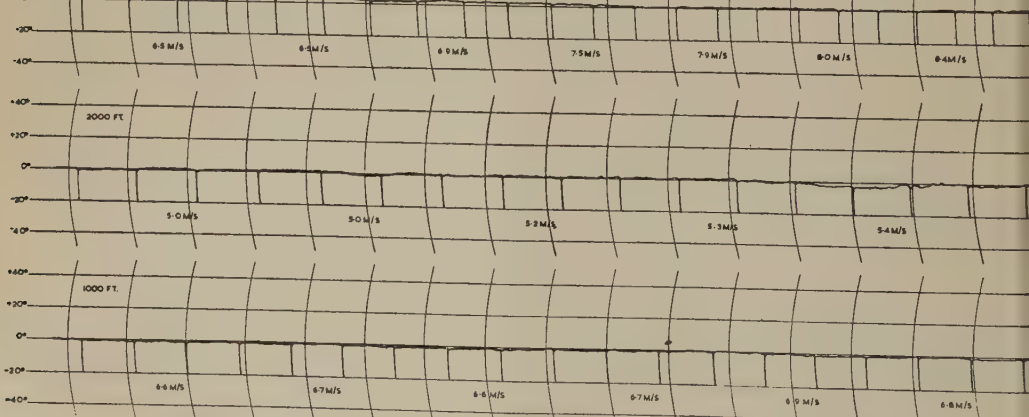


Figure 5. Sample record of wind inclination at 1,000, 2,000 and 3,000 ft, at Cardington, 28 June 1956, ahe of a warm front, weather overcast, with cloud base above 3,000 ft, and a slight inversion in the lowest 1,000

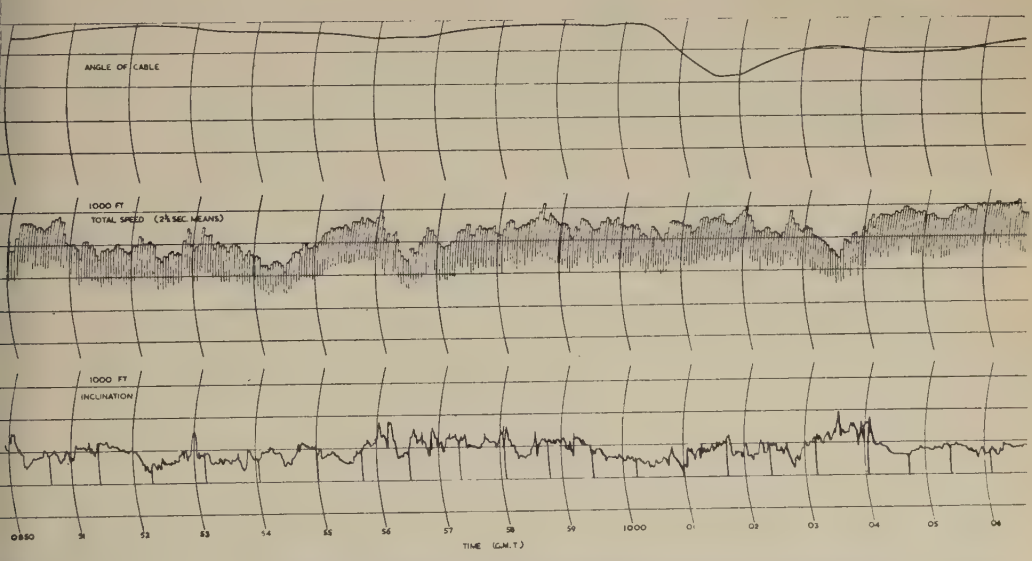
ground equipment could thus be made after any length had been paid out. Generally, dings of temperature were taken every 250 ft during ascent and descent and the nperature structure was sampled at the rate of 1,000 ft in about 10 min. The total nd-speed airmeter, described in Section 2 (b), was attached to the balloon cable just ow the lowest gustiness meter.

Samples of some of the records taken are shown in Figs. 4, 5 and 6. The heights oted on the gustiness records are cable lengths, but can be regarded as the height ve ground to within 3 per cent. Accuracy of the gustiness records is governed by the gree of balloon movement which took place during the recordings.

Fig. 4 shows wind inclination at 1,000, 2,000 and 3,000 ft in well-developed con- tive conditions. The wind speeds indicated at intervals on the chart are mean values rived from the reference pulses. The corrections for balloon movement are such as increase the positive inclination (balloon rising) by a few degrees for about 1 min ar 1041½ GMT, and to reduce the inclination for a period of ¾ min around 1045 GMT is example is especially noteworthy in that, for a period of approximately 3 min, which indicated on the chart, the top instrument was actually in a large cumulus cloud. nother interesting feature is the apparent lack of turbulence at 2,000 and 3,000 ft soon er the passage of the cloud.

By way of contrast, Fig. 5 shows wind inclination recorded at the same heights ring the night, in overcast conditions, ahead of a warm front. No measurable error e to balloon movement was evident during this period. This record is noteworthy showing almost complete absence of turbulence at all heights, despite the appreciable nd speeds (e.g., up to nearly 9 m/sec at 3,000 ft) and the substantial wind gradient in e vertical.

Finally, Fig. 6 shows an example of simultaneous records of wind inclination, total eed and angle of tethering cable. This refers to a height of 1,000 ft and to cloudy nditions with 7/8 Sc base 2,000 ft. Significant balloon movement in this example curs between 1000¼ and 1003¼ GMT. This introduces a correction over the period 000¼ to 1001½ GMT of less than 1°* in inclination and + 0·6 m/sec in speed. The rresponding corrections for the following minute are respectively less than 1° and



igure 6. Sample record of wind inclination and total speed (m/sec) at 1,000 ft and angle of tethering cable, at Cardington, 3 May 1957. Cloudy, 7/8 Sc, base 2,000 ft.

* The sign and magnitude of this correction will depend on the particular value of the apparent inclination in relation to the angle the tethering cable.

— 0·4 m/sec. It should be noted that the total speed is in histogram form, the height of each ' pulse ' representing the mean speed over the preceding period of 2½ sec. Accordingly, in correlating the speed and inclination records, the former requires displacement by 2½ sec to the left.

Acknowledgments

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References

Field, J. H. and Warden, R. 1933 *Geophys. Mem. Met. Off.*, VII, No. 59.

Ower, E. 1949 *The measurement of air flow*, London (Chapman and Hall), p. 150.

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The air flow over a ridge – the effects of the upper boundary and high-level conditions

By G. A. CORBY and J. S. SAWYER
Meteorological Office, Dunstable

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SUMMARY

The solution obtained by perturbation theory for the airflow over a ridge depends on the mathematical conditions imposed at the upper boundary of the region for which a solution is obtained. New arguments are developed as to the choice of upper-boundary condition appropriate to the atmospheric problem; these lead to the solution already adopted by Queney (1947).

In certain airstreams it is possible for lee waves to have their largest amplitude in the upper troposphere and stratosphere. The character of the airstream at high levels plays a dominant part in determining the characteristics of these waves, but it has only negligible effects on the more familiar type of lee wave which has its largest amplitude in the lower troposphere.

1. INTRODUCTION

A differential equation can be derived to represent the flow of air over a ridge. If attention is confined to the two-dimensional problem and small perturbations only are considered, the equation can be linearized and takes on a simple form.

The problem of obtaining the flow over a particular ground profile consists of determining the solution of the equation subject to suitable boundary conditions. The boundary condition at the ground is imposed directly by the requirement that the velocity there should be everywhere parallel to the ground, but additional conditions are also required to be applied either at some artificial boundaries introduced to limit the problem in the vertical and horizontal or at great heights and large horizontal distances. Only if adequate conditions of this nature are imposed does the problem become mathematically determinate.

It is usual to provide adequate boundary conditions at large horizontal distances by requiring that the disturbances to the flow should tend to zero for large distances upstream and should remain finite for large distances downstream.

In the present paper we consider what boundary conditions should be applied at great heights and whether the introduction of an arbitrary upper boundary gives a justifiable approximation to the flow. The results of some computations using model airstreams including the stratosphere are presented to illustrate the effect on the flow of different characteristics in the airstream at high levels. (In this paper the symbol $\rightleftharpoons$ is used to denote ' equals the real part of ').

2. THE PHYSICAL PROBLEM

In the treatment of the problem by Scorer (1949), the flow is defined by ψ which is the departure of the stream function from its value in the undisturbed stream, and the differential equation obtained for small perturbations in two-dimensional flow takes the form

$$\frac{\partial^2 \psi}{\partial z^2} - \left(\frac{g}{c^2} + \beta\right)\frac{\partial \psi}{\partial z} + (l^2 - k^2)\,\psi = 0 \qquad . \qquad . \qquad . \qquad (1)$$

where z = height measured upwards

g = gravity

c = velocity of sound

β = static stability in the form $\theta^{-1}\,\partial\theta/\partial z$, θ being the potential temperature

$l^2 = \dfrac{g\beta}{U^2} - \dfrac{1}{U}\dfrac{\partial^2 U}{\partial z^2}$, a function of the stability and wind profiles in the undisturbed stream

U = undisturbed horizontal wind speed

$k = 2\pi/$wavelength of the disturbance

We wish to consider the behaviour of this equation for large values of z under conditions typical of the stratosphere. It is impossible to deal with all possible profiles of wind and temperature in the stratosphere and for simplicity we shall assume that in the stratosphere U and T are constant. There is no reason to suppose that this restriction will invalidate any conclusions drawn about the nature of the upper boundary.

Then
$$\left(\frac{g}{c^2} + \beta\right) = -\frac{1}{\rho}\frac{\partial \rho}{\partial z} = \frac{g}{RT} \qquad (2)$$

Also
$$l^2 = \frac{g\beta}{U^2} = \frac{\gamma - 1}{\gamma}\frac{g}{RTU^2} \qquad (3)$$

Hence in a uniform stratosphere Eq. (1) becomes

$$\frac{\partial^2 \psi}{\partial z^2} - \frac{g}{RT}\frac{\partial \psi}{\partial z} + \left(\frac{\gamma - 1}{\gamma}\frac{g^2}{RTU^2} - k^2\right)\psi = 0 \qquad (4)$$

The solutions have the form $\psi = e^{mz}\, e^{ikx}$

where
$$m = \tfrac{1}{2}\frac{g}{RT}\{1 \pm (1 - 4M)^{\frac{1}{2}}\} \qquad (5)$$

and
$$M = \frac{R^2 T^2}{g^2}\left(\frac{\gamma - 1}{\gamma}\frac{g^2}{RTU^2} - k^2\right) \qquad (6)$$

As m may be real or complex these solutions represent waves whose amplitude increases or decreases exponentially (or is constant) with height or which vary harmonically in the vertical with increasing or decreasing (or constant) amplitude. In any given case the types of solutions which appear will depend on k, T and U.

(a) *Wholly real values of m*

We first consider the case of real roots for m. This occurs when $4M \leqslant 1$ or, in terms of the wavelength, when $k^2 \geqslant g^2/R^2\,T^2\left(\dfrac{\gamma - 1}{\gamma}\dfrac{RT}{U^2} - \tfrac{1}{4}\right)$. The solution $\psi = e^{mz}\, e^{ikx}$ represents a wave which increases in amplitude with increasing height if $m > 0$ and the wave will decrease with height if $m < 0$. The wave fronts will necessarily be vertical. There are thus three possibilities

(i) If $M > 0$, both solutions represent waves which increase in amplitude with height.

(ii) If $M < 0$, one solution increases with height and the other decreases.

(iii) If $M = 0$, one solution has constant amplitude with height and the other increases.

In terms of k, the condition $M \gtrless 0$ becomes

$$k^2 \lessgtr \frac{\gamma - 1}{\gamma} \frac{g^2}{RTU^2} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Before deciding, however, which solutions are relevant to our problem, we must ote that as the density of the air decreases with height, the energy of the wave disturbance ay decrease with height even when the amplitude increases with height. It would seem ore appropriate to exercise discrimination in terms of the energy of the disturbance an in terms of the amplitude.

The kinetic energy of the wave motion is proportional to $\rho\,(u^2 + w^2)$, where u and are the perturbation velocities in the x and z directions

e.

$$\text{K.E.} \propto \rho \left\{ \left(\frac{\partial \psi}{\partial z}\right)^2 + \left(\frac{\partial \psi}{\partial x}\right)^2 \right\}$$

$$\propto \rho\, e^{2mz}$$

$$\propto e^{(2m - g/RT)z}$$

Thus the kinetic energy of the disturbance decreases with height provided $< \frac{1}{2}\, g/RT$. Hence of those solutions under consideration, viz., real values of m, each air comprises one solution having energy increasing with height and one solution with nergy decreasing with height. The amplitude of the latter solution will itself decrease ith height only if $M < 0$. There is, therefore, a narrow band of wavelengths corres- onding to $0 < M < \frac{1}{4}$ for which one of the solutions, although having increasing mplitude with height, nevertheless has decreasing energy; but this band proves to nvolve only a very small range of wavelengths. For solutions within this band of wave- engths the amplitude will become so great at some heights that the basic assumption f perturbation theory (namely small amplitude) will become invalid. Nevertheless here is no reason to expect that, if the complete non-linear equations could be solved or these high levels, the order of magnitude of the energy would be altered. Con- equently we accept as appropriate to our problem all solutions of Eq. (1) having energy ecreasing with height. This is achieved by selecting the negative sign in the expression or m given in Eq. (5).

(*b*) *Complex values of m*

If $4M > 1$, that is for wavelengths longer than a critical limit, m will be complex nd the solutions take the form

$$\psi = e^{gz/2RT}\, e^{i(kx \pm Nz)} \propto p^{-\frac{1}{2}}\, e^{i(kx \pm Nz)} \quad . \quad . \quad . \quad (8)$$

where $N = \frac{1}{2}\, g/RT\,\{4M - 1\}^{\frac{1}{2}}$

$$= (l^2 - k^2 - \tfrac{1}{4}\, g^2/R^2\, T^2)^{\frac{1}{2}}$$

Hence $$u = \frac{\partial \psi}{\partial z} \propto \tfrac{1}{2}\, p^{-\frac{1}{2}} \frac{\partial p}{\partial z} \propto p^{-\frac{1}{2}}$$

$$w = -\frac{\partial \psi}{\partial x} \propto p^{-\frac{1}{2}}$$

The kinetic energy of the disturbance is proportional to $\rho\,(u^2 + w^2)$, i.e., to p^{-1} = constant. The kinetic energy does not, therefore, decrease with height for any avelength in the range under consideration and the total kinetic energy above any level ould appear to be infinite. However, such solutions occur only for the flow over an nfinite series of sinusoidal troughs and ridges, and it can be demonstrated that for the ow over a ridge extending only a finite distance up- or down-stream the energy bove any level is finite and tends to zero as that level is raised indefinitely. This

difference arises because, as the energy is proportional to the square of the velocity the energy at the various wavelengths in the range is not additive and the interferenc between waves of different wavelengths prevents the appearance of infinite energy.

The above discussion indicates that it is not possible to devise an adequate upper boundary condition for this range of wavelengths on the basis of energy considerations The problem remains as to which of the two solutions given by Eq. (8), or what mixtur of the two solutions, should be adopted.

We now consider the flow when a rigid upper boundary exists. This eliminate any ambiguity and renders the flow fully determinate. We shall then examine the flow as the rigid lid is removed towards infinity. If the flow at finite heights tends to a limi under these circumstances, the inference would be very strong that this limit is the appro priate choice for the unbounded atmosphere in nature.

Suppose our two-dimensional ridge is specified by the Fourier Integral

$$\zeta_0 = \int_0^\infty e^{ikx} F(k)\, dk \quad . \quad . \quad . \quad . \quad . \quad (9)$$

where $F(k)$ is real and necessarily bounded for all positive k, and let us assume that a rigid lid exists at $z = h$.

The general solution of our differential equation, as applied to a uniform airstream, is

$$\psi = e^{\frac{1}{2}\frac{g}{RT}z} (Ae^{iNz} + Be^{-iNz})\, e^{ikx} \quad . \quad . \quad . \quad (10)$$

where $N = (l^2 - k^2 - \frac{1}{4} g^2/R^2 T^2)^{\frac{1}{2}}$

and A, B are constants which may be complex.

Put $l^2 - \frac{1}{4} g^2/R^2 T^2 = L^2$. We are only concerned with the case of $L^2 > 0$, since otherwise m would not be complex for any real value of k.

By introducing the upper and lower boundary conditions into Eq. (10) we derive as the expression for the displacement of a streamline at height z in the undisturbed stream :

$$\zeta_z = \int_0^\infty e^{\frac{1}{2}\frac{g}{RT}z} (\cos Nz - \sin Nz \cot Nh)\, e^{ikx} F(k)\, dk$$

The part of the integral over the range $k > L$ corresponds to wholly real values of m, for which the upper-boundary condition has been shown to be determinate from energy considerations. We need only consider, therefore, the range $k < L$.

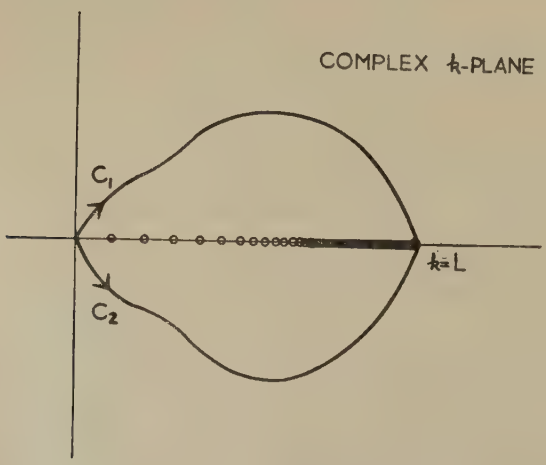

Figure 1. Contours used for contour integration.

We note that if $Lh > \pi$ the integrand has one or more poles in this range of k and as h becomes large, the number of poles increases. Thus the effect of the lid is to introduce lee waves, which become infinite in number as $h \to \infty$. It can be shown that individually they become infinitesimal in amplitude for large h except for the special case of $k = 0$ and $Lh = n\pi$, when the amplitude is indeterminate. However the shape of the mountain ridge is not sensibly altered by defining it as $\int_{\epsilon}^{\infty} e^{ikx} F(k)\, dk$, where ϵ is small and positive, and this avoids the singularity.

Following the method established by Lamb (1932) for similar lee-wave problems, we integrate by deforming the contour in the complex plane of k, noting that the integrand has no poles off the axes (see Fig. 1). That part of the displacement arising from the range $0 < k < L$ is thus given by

$$\zeta_z' = \left\{ e^{\frac{1}{2}\frac{g}{RT}z} \int_{C_1} (\cos Nz - \sin Nz \cot Nh)\, e^{ikx} F(k)\, dk + \pi i\, (\Sigma \text{ residues at the poles}) \right\} \quad \text{for } x > 0 \quad . \quad . \quad (11)$$

and

$$\zeta_z' = \left\{ e^{\frac{1}{2}\frac{g}{RT}z} \int_{C_2} (\cos Nz - \sin Nz \cot Nh)\, e^{ikx} F(k)\, dk - \pi i\, (\Sigma \text{ residues at the poles}) \right\} \quad \text{for } x < 0 \quad . \quad . \quad (12)$$

We now assume from practical experience that the disturbance of the flow must decrease to zero upstream and we therefore annul the waves upstream by adding πi (Σ residues) on both sides of the mountain, so obtaining as the practical solution in our problem

$$\zeta_z' = \left\{ e^{\frac{1}{2}\frac{g}{RT}z} \int_{C_1} (\cos Nz - \sin Nz \cot Nh)\, e^{ikx} F(k)\, dk + 2\pi i\, (\Sigma \text{ residues at the poles}) \right\} \quad \text{for } x > 0 \quad . \quad . \quad (13)$$

and

$$\zeta_z' = \left\{ e^{\frac{1}{2}\frac{g}{RT}z} \int_{C_2} (\cos Nz - \sin Nz \cot Nh)\, e^{ikx} F(k)\, dk \right\} \quad \text{for } x < 0 \quad . \quad . \quad (14)$$

We must now consider whether this integral tends to a finite limit as $h \to \infty$. Let $k = k_1 + ik_2$ along contour C_1

i.e. $$\{L^2 - (k_1 + ik_2)^2\}^{\frac{1}{2}} = N = N_1 + iN_2, \text{ say,}$$

or $$N_1^2 - N_2^2 = L^2 - k_1^2 + k_2^2$$

and $$N_1 N_2 = -k_1 k_2$$

Now $N = +L$ when $k_1 = k_2 = 0$ and N_1 and N_2 must be of opposite sign. Thus in the first quadrant N_1 is positive and N_2 is negative. Hence $\cot Nh$ is of the form $\cot (N_1 - iN_2)\, h$ along C_1.

It can be shown that

$$\lim_{h\to\infty} \{\cot (N_1 - iN_2)\, h\} = i$$

Hence* $$\lim_{h\to\infty} \int_{C_1} (\cos Nz - \sin Nz \cot Nh)\, e^{ikx} F(k)\, dk = \int_0^L e^{-iNz + ikx} F(k)\, dk$$

* It is easily verified that it is here justified to take $\lim_{h\to\infty}$ inside the integral sign.

We now require to evaluate $\underset{h\to\infty}{\text{Lim}}\,(2\pi i\ \Sigma \text{ residues})$.

The residue at a pole where $k = k'$, $N = N'$ is :

$$\frac{N'}{hk'}(\sin N' z)\, e^{ik'x} F(k')$$

The poles occur where $Nh = 0,\ \pi,\ 2\pi \ldots\ldots n\pi$

i.e., where $k = (L^2 - r^2\pi^2/h^2)^{\frac{1}{2}}$

$r = 0,\ 1,\ 2 \ldots\ldots n$

Put $Lh = \pi(n + \alpha)$ where $0 \leqslant \alpha < 1$.

Then the poles occur where $k = L\{1 - r^2/(n + \alpha)^2\}^{\frac{1}{2}}$

Hence $\underset{h\to\infty}{\text{Lim}}\ \Sigma$ residues

$$= \underset{n\to\infty}{\text{Lim}} \left\{ e^{\frac{1}{2}\frac{g}{RT}z} \sum_{r=0}^{n} e^{ik'x} F(k') (\sin N' z) \left(\frac{rL}{\pi (n + \alpha)^2 \{1 - r^2/(n + \alpha)^2\}^{\frac{1}{2}}} \right) \right\}$$

$$= \frac{1}{\pi} e^{\frac{1}{2}\frac{g}{RT}z} \int_0^L e^{ikx} F(k) \sin Nz\, dk$$

Thus $2\pi i\ (\Sigma \text{ residues}) = 2ie^{\frac{1}{2}\frac{g}{RT}z} \int_0^L e^{ikx} F(k) \sin Nz\, dk$

Substituting in Eqs. (13) and (14) we see that

$$\zeta_z' = e^{\frac{1}{2}\frac{g}{RT}z} \int_0^L e^{+iNz+ikx} F(k)\, dk \qquad . \qquad . \qquad . \qquad (15)$$

It would appear, therefore, that the proper solution is obtained by taking $B = 0$ in the general solution, Eq. (10). The same conclusion was reached by different methods by other authors, e.g. Queney (1947), Palm (1952) although Scorer (1949) adopted the choice $A = 0$. In a later paper (Scorer 1954) he reviewed the implications of the devices which have been or might be invoked to resolve the upper-boundary problem and considered that the problem might be an unreal one which arises because of the method of formulation of the problem.

The present treatment can be interpreted physically as follows. If there is a rigid upper boundary, then the flow of a uniform airstream over a ridge comprises a symmetrical disturbance over the ridge plus a large number of lee waves having wavelengths in the range $2\pi/L$ to ∞. The waves tend to interfere away from the ridge and as the lid is removed to infinity they annul completely for large x. They contribute, however, a finite disturbance in the neighbourhood of the ridge for positive x and the whole flow tends to a limit which is asymmetrical. It seems that this must be the solution appropriate to the unbounded atmosphere.

It is of interest to consider briefly the nature of the solutions obtained for a two-layer model in which the value of L is constant in the lower layer and zero in the upper layer. Under these conditions m is not complex in the upper layer for any value of k; accordingly there is no indeterminacy at high levels. As the depth of the lower layer is increased, more and more lee waves appear for values of k in the range $0 < k < L$ and these become infinite in number as the interface is removed to infinity. Once again the flow at finite heights tends to the asymmetrical limit corresponding to the choice $B = 0$ in Eq. (10). The proof is somewhat more lengthy but the artificiality of a rigid upper boundary is avoided.

One further aspect is worthy of mention here. In the following paragraphs we derive quations for the flow in a two-layer model airstream (Eq. (17) and (18)). If the upper ayer is the more stable, i.e., has the larger value of l^2, then the integrand in Eq. (17) can ave no real poles and accordingly no infinite train of lee waves could appear. This is onsistent with the lee-wave theory of Scorer (1949) in that his condition for the occurrence f an infinite train of lee waves in the two-layer airstream could never be satisfied with l^2 reater in the upper layer than in the lower. Nevertheless, it can be shown that when he upper of the two layers has the larger value of l^2, it is possible for poles to occur for omplex values of k and precise conditions for these can be formulated, the conditions eing closely analogous to those deduced by Scorer (1949) for the more familiar real oles. When they occur, the complex poles are necessarily displaced from the real k xis into the first quadrant of the complex k plane if the upper-boundary condition $B = 0$ s used and to the fourth quadrant if $A = 0$. The lee waves corresponding to such omplex poles comprise a finite train, their amplitude decreasing exponentially away rom the mountain; the more stable the upper layer is compared with the lower layer, he more slowly does the amplitude decrease away from the mountain. Since the poles re displaced from the real axis the finite train of waves is necessarily confined to one side f the mountain and, if the upper-boundary condition advocated here is adopted, lies ntirely on the downstream side. The opposite choice results in ' lee ' waves on the pstream side. These considerations constitute an additional argument in favour of the hoice $B = 0$ in Eq. (10). This method of discrimination is similar in some respects o the method depending on the introduction of Rayleigh friction although the latter nethod has been criticised on the grounds that the form of the frictional term used is omewhat artificial.

3. The characteristics of certain model airstreams

In studying the character of the flow with specific model airstreams we shall use the impler form of Eq. (1) as employed by Scorer (1949), viz :

$$\frac{\partial^2 \psi}{\partial z^2} + (l^2 - k^2)\,\psi = 0 \quad . \quad . \quad . \quad . \quad (16)$$

If we wished we could follow Queney (1947) and allow approximately for the neglected niddle term by multiplying the displacements of the streamlines deduced from Eq. (16) by the factor $(\rho_0/\rho)^{\frac{1}{2}}$ where ρ is the density and suffix 0 refers to the ground.

Suppose our model airstream comprises layers of depth $a_1, a_2, \ldots$. in each of which is constant, the values of l being $l_1, l_2, \ldots$. and the suffixes being numbered from the ground upwards.

Then, if we adopt the upper-boundary condition established above, the flow over he mountain profile given by

$$\zeta_0 = \int_0^\infty e^{ikx} F(k)\, dk$$

akes the form

$$\zeta_z = \int_0^\infty (\cos \nu_1 z + f(k) \sin \nu_1 z)\, e^{ikx} F(k)\, dk \quad . \quad . \quad (17)$$

where

$$\nu_1 = + (l_1^2 - k^2)^{\frac{1}{2}}$$

The flow specified by Eq. (17) applies to the lower layer and for convenience we have ssumed that the undisturbed wind speed U is constant with height, i.e., that the variations f l with height arise solely owing to variations of static stability*. The function $f(k)$ akes the following forms for two, three and four-layer models :

* If we permitted U to vary with height, the displacements of the streamlines subsequently computed would be multiplied by he factor U_0/U_z.

two-layer

$$f(k) = \frac{f_1}{f_2} = \frac{\nu_1 t_1 + i\nu_2}{\nu_1 - i\nu_2 t_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

three-layer

$$f(k) = \frac{f_1}{f_2} = \frac{\nu_2(\nu_1 t_1 + \nu_2 t_2) + i\nu_3(\nu_2 - \nu_1 t_1 t_2)}{\nu_2(\nu_1 - \nu_2 t_1 t_2) - i\nu_3(\nu_1 t_2 + \nu_2 t_1)} \quad . \quad . \quad . \quad (19)$$

four-layer

$$f(k) = \frac{f_1}{f_2} = \frac{\nu_2(\nu_3 - i\nu_4 t_3)(\nu_1 t_1 + \nu_2 t_2) + \nu_3(i\nu_4 + \nu_3 t_3)(\nu_2 - \nu_1 t_1 t_2)}{\nu_2(\nu_3 - i\nu_4 t_3)(\nu_1 - \nu_2 t_1 t_2) - \nu_3(i\nu_4 + \nu_3 t_3)(\nu_1 t_2 + \nu_2 t_1)} \quad (20)$$

where $$\nu_r = +(l_r^2 - k^2)^{\frac{1}{2}}$$

and $$t_r = \tan(\nu_r a_r)$$

The values of k giving rise to lee waves are the roots of the equations $f_2 = 0$ and in the lower layer, that part of the displacement of a streamline due to the lee waves appropriate to a root $k = k^*$ is given by :

$$\zeta_z \rightleftharpoons 2\pi i \left[\frac{f_1}{\partial f_2/\partial k} \sin \nu_1 z \, e^{ikx} F(k)\right]_{k=k^*} \quad . \quad . \quad . \quad (21)$$

Similar expressions can be derived for a multilayer model in which a lid exists at any specified level.

Eqs. (18-21) can be used to compute the properties of the lee waves which arise in any two, three or four-layer model airstream, the awkward quantity $\partial/\partial k\,(f_2)$ being obtained numerically in the course of the computations. The behaviour in the vertical of the lee waves in the layers above the lowest is obtained by applying considerations of continuity at the interfaces.

(a) *Two-layer airstream*

We shall first consider a simple two-layer airstream of the type which can contain lee waves and then examine how these lee-wave characteristics are modified as different conditions are specified at high levels. We take a lower stable layer, 1 km deep, in which $l^2 = 4{\cdot}0\ \text{km}^{-2}$ and suppose that in the upper layer which extends to infinity $l^2 = 0{\cdot}3\ \text{km}^{-2}$. The appropriate lee-wave equation has only one real root viz : $k = 0{\cdot}76\ \text{km}^{-1}$, corresponding to a lee wavelength of 8·3 km.

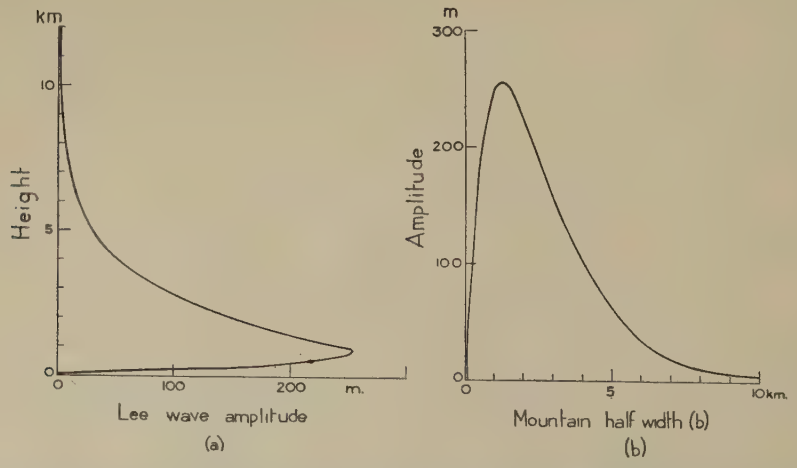

Figure 2. The properties of the lee wave ($k = 0{\cdot}76\ \text{km}^{-1}$) for a two-layer airstream extending to infinity and flowing over a ridge of height 100 m. (a) the variation of amplitude with height for the optimum-width ridge. (b) the variation with mountain width of the maximum amplitude in the lower layer.

In order to assign an absolute magnitude to the amplitude we specify the mountain rofile as that given by

$$\zeta_0 = hb \int_0^\infty e^{ikx-kb}\, dk \quad . \quad . \quad . \quad . \quad . \quad (22)$$

nd we take the height of the ridge, h, as 0·1 km. The amplitude of the lee wave will vary ith b, the so-called ' half-width ' parameter, and to particularise we take $b = k^{-1}$ being ne value which excites the largest-amplitude lee waves. In a discussion of the amplitude f lee waves, Corby and Wallington (1956) regarded the mountain having $b = k^{-1}$ as ne optimum width for the particular lee wavelength, and likened the effect near this alue to resonance.

Under these conditions the variation with height of the lee-wave amplitude as deter- nined from Eq. (21) is that illustrated in Fig. 2 (a). The variation of amplitude with height ollows the usual pattern for such an airstream in that a maximum amplitude is attained n the lower layer, where the amplitude variation in the vertical is sinusoidal; in the pper layer the amplitude falls off exponentially. For convenience, the value of the lee- ave amplitude at the level where the maximum is achieved is illustrated for other values f b in Fig. 2 (b).

It is of interest to consider here the effect of introducing a rigid lid at, say, 10 km bove the interface or roughly at tropopause level. It is found that the lee wave at $= 0{\cdot}76$ km^{-1} still appears, without sensible change in its properties, but that there is n additional lee wave at $k = 0{\cdot}415$ km^{-1}. The variation with height of the two waves s indicated in Fig. 3 (a) where the amplitude of each is depicted for its own optimum- idth mountain. Fig. 3 (b) enables the amplitudes for any particular mountain to be ompared. The second wave, due to the presence of the lid, is noteworthy in that it has ts maximum amplitude in the upper layer. This and several other computations suggest hat, although the imposition of a lid at great height is theoretically a useful artifice, it would ot give a good approximation to the flow of an unbounded atmosphere unless imposed t very great height indeed. In the present case, with the lid at height 11 km, only one dditional lee wave is induced and clearly we should require a sufficiently large number o annul almost completely by interference away from the mountain. The point would be elevant if one considered using a rigid lid as an upper-boundary condition when treating he problem by numerical integration of the differential equation for the flow, or when onsidering model experiments.

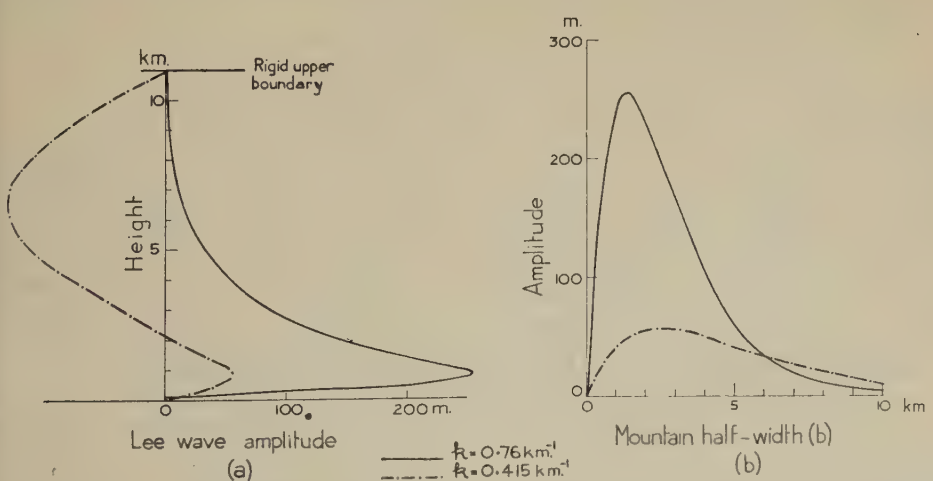

Figure 3. The properties of the two lee waves for a two-layer airstream bounded by a rigid lid and flowing ver a ridge of height 100 m. (a) The variation of amplitude with height, each lee wave being associated ith its optimum-width ridge. (b) The variation with mountain width of the maximum amplitudes in the lower layer.

(b) *Four-layer airstream*

We now consider a four-layer airstream in which the two lower layers of dept 1 and 10 km have the same values of l^2 used in the two-layer example. They are surmounte by a third layer of depth 30 km in which $l^2 = 6{\cdot}0\ \text{km}^{-2}$; this is intended to represent stable stratosphere. Above this we assume the lapse rate to be adiabatic ($l^2 = 0$). Such model is not completely unrealistic, for the upper half of the ozonosphere is believe to be a region of active convection (see Scorer 1953). Furthermore, the presence of th upper layer with $l^2 = 0$ means that the disturbance necessarily decreases with height i that layer over the whole range of wavelengths.

As the terms involving $\tan(\nu_3 a_3)$ in Eq. (20) oscillate rapidly for changing values o k, the wavelength equation $f_2 = 0$ has a large number of roots. Indeed, it is found tha there are 26 nodal surfaces in the solution for $k = 0$, indicating the existence of 26 le wavelengths. These can be conveniently put into three groups, viz., those in the range $0 < k < l_2$, $l_2 < k < l_1$ and $l_1 < k < l_3$. No lee waves occur for $k > l_3$. All the le waves have sinusoidal variation of amplitude through the stable stratosphere and ex ponential decrease of amplitude in the uppermost layer.

There are two lee waves in the first group ($0 < k < l_2$). These are of appreciabl amplitude at low levels and have even larger amplitude in the upper troposphere an stratosphere, especially if we bear in mind the additional factor $(\rho_0/\rho)^{\frac{1}{2}}$ which for simplicit has been omitted at this stage. Formally the solutions of the equation $f_2 = 0$ correspondin to these lee waves arise because of the decrease in stability between the second and uppe layers and they would not appear, for example, if we made $l_4 = l_2$. However, som similarity will be noted between these waves and the solution which was introduced int the two-layer model by the rigid lid at 11 km, and it can be shown that the form of thes solutions in the lower layers is largely determined by the large increase of stability i passing from the second to the third layer, corresponding to the increase in stability a the tropopause. Although, if no less stable layer is introduced at great heights no infinit trains of lee waves appear as solutions, nevertheless $|f(k)|$ in Eq. (17) becomes large fo

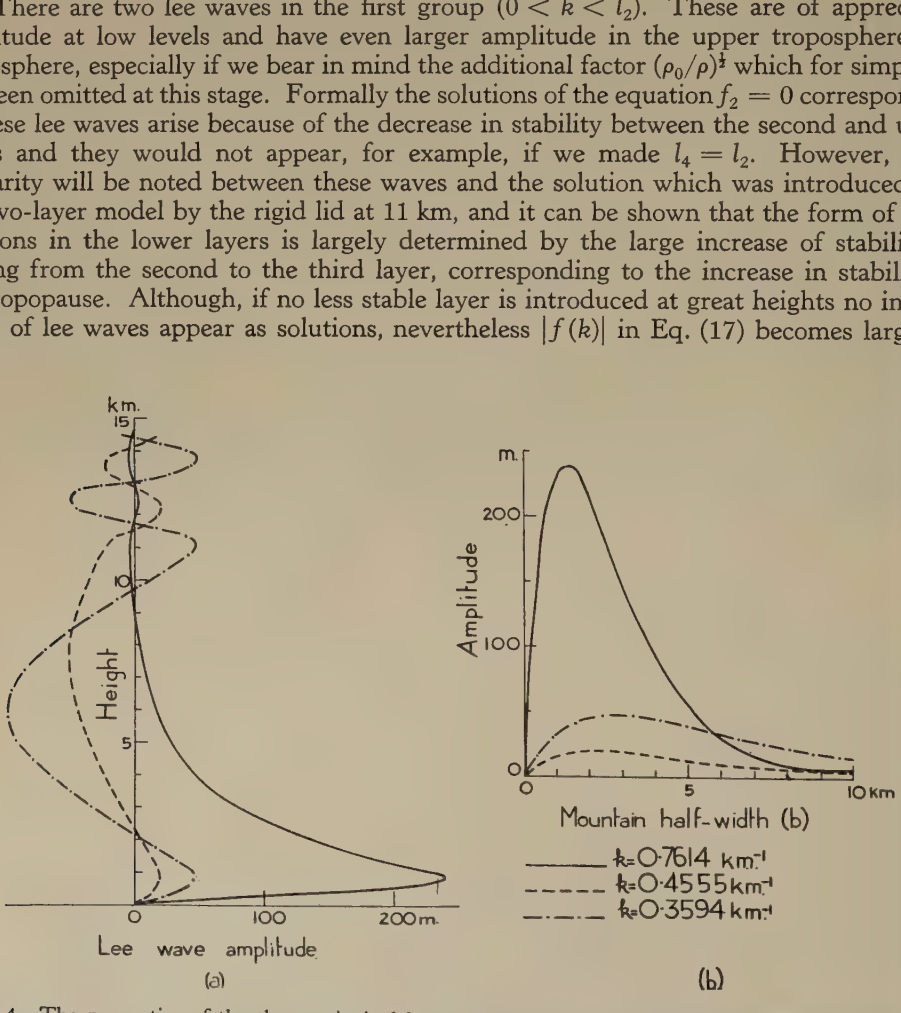

Figure 4. The properties of the three principal lee waves for a four-layer airstream extending to infinit and flowing over a ridge of height 100 m. (a) the variation of amplitude with height, each lee wave bein associated with its optimum-width ridge. (b) the variation with mountain width of the amplitudes in th lower layer.

values of k near that corresponding to lee waves with a lid and it can be shown that oscillations of the same type appear in the solution downstream of the obstacle but that these die away slowly downstream.

The first root in the second group, viz., that having the smallest k, occurs very nearly at $k = 0{\cdot}76$ km^{-1} and the amplitude variation through the troposphere is very closely similar to that of the wave obtained from the two-layer model; indeed for practical purposes the waves can be regarded as identical. The next wave of the group has an amplitude in the lower layer only about 1/1,000 of that at $k = 0{\cdot}76$ km^{-1} and subsequent waves have even smaller amplitudes. The waves in the third group are also of negligible amplitude. Many of these additional waves do, however, attain their maximum amplitude in the stratosphere although even there the absolute values are negligible.

The amplitude and its variation with height are illustrated for the three main waves in Fig. 4 (a). As before, the absolute magnitude of the amplitude is depicted in each case for the appropriate optimum-width mountain, and Fig. 4 (b) may be used to compare the amplitudes in the lower layer for any other specific mountain width.

If the upper layer is omitted and a lid is imposed at the top of the stratosphere, once again the main lee wave near $k = 0{\cdot}76$ km^{-1} appears without significant change in its properties. There are also two lee waves of longer wavelength (smaller k) having similar properties to those with the four-layer model, whilst all the other lee waves have negligible amplitude at all heights.

For most purposes the lee waves having the greatest interest are those which attain their maximum amplitude in the lower troposphere and which die away upwards. The unbounded two-layer airstream first considered had one lee wavelength of this type and the subsequent computations suggest that, provided the same tropospheric conditions are maintained, this lee wave appears without perceptible change in its properties whatever conditions are assumed at high levels. Thus lee waves of this type do not appear to be at all sensitive to changes in the wind or stability profiles at high levels. That this is not a fortuitous result arising because of the particular l^2 profile adopted, but is true in general, can be deduced by a study of Eq. (20). It also appears that lee waves may arise as a result of the change of stability in the region of the tropopause. These are similar to the solutions obtained if a rigid lid is imposed at a level corresponding to the tropopause but for proper representation of these the stability at great heights must be considered. The introduction of a layer of neutral stability or a lid at a very high level is a useful artifice to make these lee waves appear as infinite wave trains rather than wave trains of finite extent, thereby making the mathematical treatment easier.

4. The solution obtained for $k = 0$

The device of using the number of nodal surfaces in the solution for $k = 0$ as a means of determining the number of lee wavelengths was referred to earlier. We will mention another possible application of this solution.

In the problem of orographic rainfall Sawyer (1956) deduced that the smaller-scale gravity waves with wavelengths of the order of 10 km or less are largely irrelevant; our concern must be with those waves comparable in length with the horizontal scale of a barrier such as the Welsh mountains. On the other hand such a barrier is still small enough to allow the neglect of oscillations about the geostrophic flow without changing the nature of the problem. Accordingly, the main features of the flow will be adequately represented by Eq. (16). Here l^2 varies from 0 to 10 km^{-2} or more; zero values occur when the atmosphere has neutral static stability, and the more commonly occurring lapse rates above the surface layers usually lead to values of $l^2 > 0{\cdot}25$ km^{-2}.

If we consider the mountain ridge whose profile is represented by Eq. (22) then clearly the amplitude of the Fourier components becomes small (e.g., $< 1/12$ of the maximum) for $k > 2{\cdot}5/b$. Thus for a mountain half-width of 20 km the contribution of components with $k > 0{\cdot}12$ is small and we are primarily concerned with k^2 in the range

0 to 0·015. It follows that a general idea of the large-scale flow can be obtained by putting $k = 0$ in Eq. (16) and solving it in the form

$$\frac{\partial^2 \psi}{\partial z^2} + l^2 \psi = 0 \qquad . \qquad . \qquad . \qquad . \qquad . \qquad (23)$$

where l is now a function of z.

Since the solution near the ground must be independent of the value of l at great heights we consider a case in which l is constant above a specified upper level. We require a solution of the form

$$\psi = F(z)\, e^{ikx} \qquad . \qquad . \qquad . \qquad . \qquad . \qquad (24)$$

where $F(z)$ is a complex function of z $\{F(z) = F_1(z) + i\, F_2(z)\}$ and in order that this should have the correct form in the upper layer with constant l, we require that, at any level in that layer, $\partial F/\partial z = ilF$. This provides the upper-boundary condition.

The solution (24) may be assumed to hold approximately throughout the range of k in which we are interested. Then the flow at level z over a mountain with profile :

$$\zeta_0 = hb \int_0^\infty e^{ikx - kb}\, dk$$

is given by

$$\zeta_z = hb \int_0^\infty (F_1 + iF_2)\, e^{ikx - kb}\, dk$$

or

$$\zeta_z = \frac{hb}{b^2 + x^2}(bF_1 - xF_2) \qquad . \qquad . \qquad . \qquad . \qquad (25)$$

The approximation implied by putting $k = 0$ in this way was used by Scorer (1949) when obtaining the part of the flow above the hill (as distinct from the lee waves). The point being made here is that it is a satisfactory and realistic approximation when studying the large-scale character of the flow over a barrier such as the Welsh mountains or the Pennines in connection with orographic rainfall. In the case of l constant with height, both the symmetric and asymmetric components in the displacement Eq. (25) have a vertical wavelength of $2\pi/l$. We, therefore, require a small value of l if the upslope motion on the windward slope is to continue as an upslope motion to considerable heights. If the air mass is very stable (statically) l will be large and we should expect to find descending air at no very great height above the windward slope. These aspects bear intimately on the problem of what conditions favour heavy orographic rain.

Although only approximate, Eq. (25) gives a specific indication of the vertical variation and asymmetry of the flow over a barrier such as the Pennines or the Welsh mountains. In layers with static stability a measurable disturbance of the temperature field would be expected and some observable effects should be apparent in the distribution of cloud layers. Horizontal flights by aircraft well above such a barrier might therefore produce direct evidence of the validity of the treatment and in particular of the appropriateness of the upper boundary condition which has been deduced and adopted.

5. Conclusions

In the theory of the airflow over mountains, further light is thrown on the problem of what choice of upper-boundary condition should be adopted by studying the limit to which the flow tends as a rigid lid, or some other upper condition which renders the problem fully determinate, is removed to infinity. This device strongly suggests that, provided we are right in assuming that a stationary solution does exist, then in the uppermost layer and over the range of wavelengths for which solutions with inclined wave fronts occur, the proper procedure is to reject the solutions giving wave fronts inclined

the downstream direction and to retain those inclined in the upstream direction. This conclusion is consistent with that reached by including small friction to achieve determinacy by obtaining the asymptotic limit of a time-varying solution.

When calculating the character of the flow in actual cases, the familiar type of lee wave which has a maximum amplitude in the lower troposphere, and which dies away above, is not at all sensitive to the conditions assumed at high levels. It would appear, however, that lee waves which have their largest amplitude in the upper troposphere and stratosphere can occur in certain airstreams depending primarily on the change of stability near the tropopause; these are of somewhat longer wavelength, and for their proper study require consideration of stability and wind in the stratosphere.

6. Acknowledgment

We are indebted to the Director-General of the Meteorological Office for permission to publish this paper.

References

Corby, G. A. and Wallington, C. E. 1956 'Airflow over mountains – the lee-wave amplitude,' *Quart. J. R. Met. Soc.*, **82**, p. 266.

Lamb, H. 1932 *Hydrodynamics*, 6th Edn., Arts. 242-246.

Palm, E. 1952 'On the formation of surface waves in a fluid flowing over a corrugated bed and on the development of mountain waves,' *Astrophys. Norvegica*, **5**, No. 3.

Queney, P. 1947 'Theory of perturbations in stratified currents with applications to airflow over mountain barriers,' *Univ. Chicago Misc. Rep. Met. Dept.*, No. 23.

Sawyer, J. S. 1956 'The physical and dynamical problems of orographic rain,' *Weather*, **11**, p. 375.

Scorer, R. S. 1949 'Theory of waves in the lee of mountains,' *Quart. J. R. Met. Soc.*, **75**, p. 41.

1953 'Air motion in the upper atmosphere.' *Nature*, **172**, p. 138.

1954 'Theory of airflow over mountains : III – airstream characteristics,' *Quart. J.R. Met. Soc.*, **80**, p. 417.

A numerical study of the wavelength and amplitude of lee wav

By C. E. WALLINGTON and JANET PORTNALL
Meteorological Office, Dunstable

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SUMMARY

Wavelengths and variation of wave amplitude with height are computed for 19 airstreams in whi lee waves have been observed. Comparison between the computed and observed wavelengths suggests th the tropospheric flow is sufficiently independent of conditions in the stratosphere to allow reasonable latitu in making a boundary assumption at some high level.

1. INTRODUCTION

In a number of papers Scorer (1949 and 1953) has discussed the occurrence of le waves in the atmosphere and has shown how the values of possible wavelengths may b calculated for an airstream which can be divided into two or more layers each with constan value of a parameter depending on wind and temperature structure. The calculation become rather lengthy if the number of layers exceeds four and it is not clear how muc the criteria for lee waves and their wavelength is dependent upon the rather artificia division of the atmosphere into only a few discrete layers.

It is therefore of interest to calculate the wavelength of lee waves in a manner which convenient for high-speed computing and which includes a more detailed treatment o the wind and temperature structure of any given airstream.

This study is restricted to steady, two-dimensional flow in a vertical plane and as th temperature and wind at great heights are not normally known it is necessary to mak some artificial boundary assumptions at the upper limit of the atmospheric layer considered Our main object in this investigation is to study the effect of these assumptions on th computed wave flow and to test the general applicability of the method by comparin observed and computed lee wavelengths.

2. THE EQUATION OF FLOW

We shall regard the flow as represented by small stationary disturbances imposed on a steady, horizontally uniform stream. Treating the flow as two dimensional in a vertical plane, an equation can be formed to determine the pressure disturbance, p, a any point on any streamline. Then assuming that hydrostatic equilibrium is maintained a differential equation defining the flow pattern can be obtained.

For continuity along a stream tube the horizontal velocity component, u, is related to its undisturbed value, U, by the equation

$$u\delta(P+p) = U\delta P \qquad (1$$

where P is the undisturbed pressure on a streamline, δP is the vertical pressure change across the stream tube in the undisturbed part of the flow and $\delta(P+p)$ is the correspond- ing vertical pressure change in the disturbed part of the flow. Thus u may be given by

$$u = U\left(1+\frac{\partial p}{\partial P}\right)^{-1} \qquad (2$$

The vertical velocity, w, may be obtained by noting that

$$u\delta z = w\delta x \qquad (3$$

where x is the horizontal coordinate and z denotes the height of the streamline through

the point being considered. If we use P as our independent variable in the vertical and substitute for u, Eq. (3) may be written in the form

$$w = U\left(\frac{\partial z}{\partial P}\right)\left(\frac{\partial p}{\partial x}\right)\left(1+\frac{\partial p}{\partial P}\right)^{-2} \quad . \quad . \quad . \quad (4)$$

For steady motion on a scale small enough to justify neglect of the Coriolis force the vertical component of the equation of motion becomes

$$\rho U\left(\frac{\partial w}{\partial x}\right) = -\frac{\partial (P+p)}{\partial z} + \rho g \quad . \quad . \quad . \quad (5)$$

where ρ denotes density and g denotes acceleration due to gravity. This equation may be rearranged into the form

$$\rho\frac{\partial z}{\partial P} = -\left(1+\frac{\partial p}{\partial P}\right)\left(g+u\frac{\partial w}{\partial x}\right)^{-1} \quad . \quad . \quad . \quad (6)$$

The second term in the denominator of the right-hand side of this equation represents the vertical acceleration of the air and is very small compared with g. Therefore an approximate form of the equation is

$$\frac{\partial z}{\partial P} = -\left(\frac{1}{\rho g^2}\right)\left(1+\frac{\partial p}{\partial P}\right)\left(g-u\frac{\partial w}{\partial x}\right) \quad . \quad . \quad . \quad (7)$$

For adiabatic flow of unsaturated air, scalar multiplication of the vector equation of motion by the velocity vector leads to Bernouilli's equation which may be expressed as

$$\rho g z + \tfrac{1}{2}\rho\,(u^2+w^2) + \left(1-\frac{1}{\gamma}\right)^{-1}(P+p) = \rho\psi \quad . \quad . \quad (8)$$

γ being the ratio of the specific heat of dry air at constant pressure to that at constant volume and ψ being a function of P only.

Differentiating Eq. (8) with respect to P and introducing the potential temperature, θ, referred to a standard pressure level, P_0, we get

$$\tfrac{1}{2}\rho\frac{\partial}{\partial P}(u^2+w^2) + \left(\frac{\gamma}{\gamma-1}\right)\left(\frac{P+p}{\theta}\right)\frac{\partial\theta}{\partial P} + \frac{u}{g}\left(1+\frac{\partial p}{\partial P}\right)\frac{\partial w}{\partial x} = \rho\frac{\partial\psi}{\partial P} \quad . \quad (9)$$

Eqs. (2) and (4) may be used to eliminate u and w from Eq. (9) and, for small disturbances only, terms involving powers of p higher than the first may be neglected. Thus after some manipulation Eq. (9) becomes

$$\frac{\partial^2 p}{\partial P^2} + \frac{2}{U}\frac{\partial U}{\partial P}\frac{\partial p}{\partial P} + \left(\frac{R\theta P_0^{\frac{1-\gamma}{\gamma}}}{gP^{\frac{1}{\gamma}}}\right)^2\frac{\partial^2 p}{\partial x^2} - \frac{RP_0^{\frac{1-\gamma}{\gamma}}}{P^{\frac{1}{\gamma}}U^2}p\frac{\partial\theta}{\partial P} = 0 \quad . \quad . \quad (10)$$

where R is the gas constant.

3. Method of computation

In order to seek solutions of Eq. (10) which are sinusoidal in the x-direction we substitute p = the real part of fe^{ikx}, where f is a function of P only, and k is a number to be determined. Eq. (10) may then be written in the form

$$\frac{\partial^2 f}{\partial P^2} + 2C\frac{\partial f}{\partial P} + (B - Ak^2)f = 0 \quad . \quad . \quad . \quad (11)$$

where A, B and C are functions of P only.

The problem of determining the existence and wavelength of lee waves now becomes one of finding values of k (= K, say) which permit solutions of Eq. (11) satisfying appropriate boundary conditions. For numerical integration of the equation it is convenient to

deal with its finite difference approximation which, after some rearrangement of terms, may be written

$$f_{p+1} = \{(n^2 A_p k^2 - n^2 B_p + 2) f_p + (nC_p - 1) f_{p-1}\} (nC_p + 1)^{-1} \quad . \quad (12)$$

where the suffixes refer to pressure levels, n being the pressure interval between successive levels.

The lower boundary condition is taken to be $f = 0$ at the 1,000-mb pressure level, but it is difficult to justify a definite assumption for the pressure disturbance at some upper boundary. Therefore we take as the upper-boundary conditions a unit pressure disturbance at the 200 mb level and make arbitrary experimental assumption of the value of f at 150 mb. Since we do not, at this stage, define a unit pressure disturbance, these assumptions involve only the ratio of the pressure disturbance at 150 mb to that at 200 mb. We shall denote this ratio by ϕ.

In the present series of computations n was given the constant value of 50 mb and the coefficients A_p, B_p and C_p were calculated for 50 mb pressure intervals centred at 200, 250, 300, ... 950 mb.

For each assumed value of ϕ and for values of $k = 0, 0{\cdot}0337, 0{\cdot}0675, 0{\cdot}1012, \ldots\ldots$ $4{\cdot}3180\ \text{km}^{-1}$ (these values happened to be convenient for the computational technique used) Eq. (12) was used to compute f at 50 mb steps from 250 mb down to the 1,000 mb level. Changes of sign for successively computed values of f at 1,000 mb showed where to intensify the search for wave numbers which allowed $f = 0$ at 1,000 mb. These wave numbers were then calculated to within $3{\cdot}239 \times 10^{-6}\ \text{km}^{-1}$. The search for values of K was not extended to wave numbers greater than $4{\cdot}318\ \text{km}^{-1}$ since experience and theory suggest that short lee waves are usually of small amplitude.

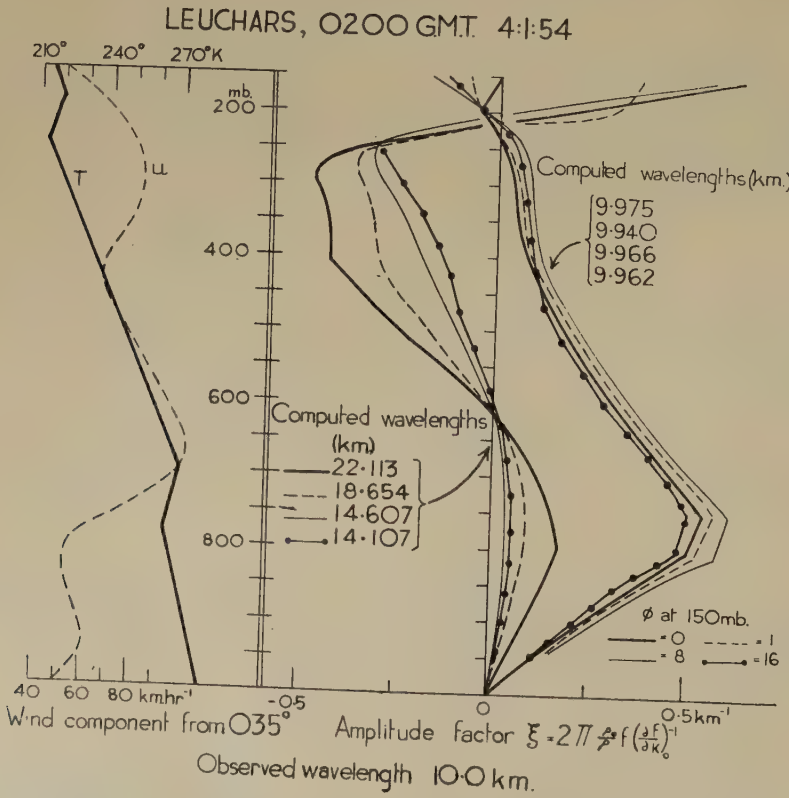

Figure 1 (see also Figs. 2 and 3). Wind and temperature distribution with height is indicated in the left of the figures. ξ curves and their associated wavelengths for several values of ϕ are shown on the right. The observed wavelengths were measured by Corby (1957).

Having computed values of K and corresponding profiles of f it was desirable to assign realistic magnitudes to the pressure disturbance.

Scorer (1949) has derived an expression for the lee-wave flow downwind of an 'idealized' two-dimensional ridge whose height, h, at a horizontal distance, x, from the crest is given by

$$h = \frac{Hb^2}{b^2 + x^2} \quad . \quad . \quad . \quad . \quad . \quad (13)$$

where H is the height of the ridge and b is a width parameter. Adapting his method to this study we can determine the flow pattern through lee waves induced by such a ridge. The vertical displacement, ζ_z, of a streamline from its undisturbed level at height z is given by the equations

$$\left.\begin{aligned} \zeta_z &= -Hbe^{-Kb}\,\xi_z \sin Kx \\ \xi_z &= 2\pi\left(\frac{\rho_0}{\rho}\right) f_z \left(\frac{\partial f}{\partial K}\right)_0^{-1} \end{aligned}\right\} \quad . \quad . \quad . \quad (14)$$

where the suffix o refers to the lower-boundary level. Values of the lee-wave amplitude factor, ξ_z, have been computed and are discussed in the following section.

4. Discussion of results

The data chosen for numerical study comprised 19 sets of Leuchars radio-sonde ascents in which Corby (1957) had already detected the effect of pronounced lee-waves.

Fig. 1 typifies many of the results obtained. Distribution of ξ with height are shown for four of the assumed upper-boundary conditions. The full lines show that with $\phi = 0$ two values of K were obtained ; the corresponding wavelengths ($2\pi/K$) are given in the

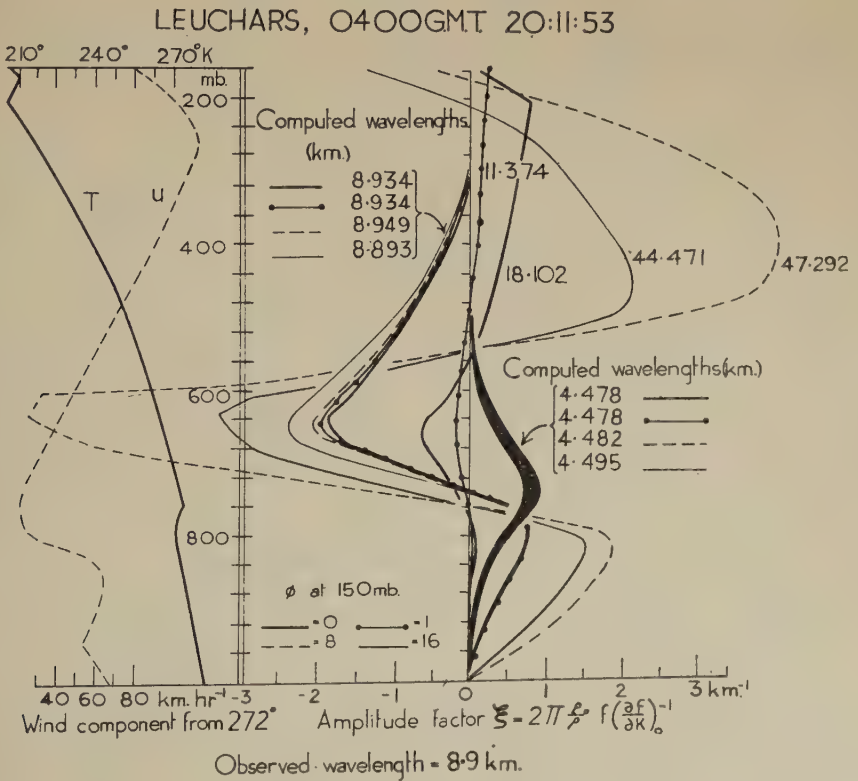

Figure 2 (see Fig. 1 for explanation).

figure. For a wavelength of 9·975 km the amplitude factor, ξ, attains a maximum o 0·57 km^{-1} in the middle of the stable layer indicated by the observed temperature profile The longer wavelength of 22·113 km appears to be associated with a significant lee-wav which has a nodal surface at 610 mb. The remaining three pairs of curves relate to $\phi = 1$ 4, and 16. A feature of the ξ curves and the computed wavelengths is that, for the shor wavelength, they change only slightly with the upper-boundary variation, whereas the long-wave value and its amplitude curve were much more sensitive to the choice of ϕ Particular attention is called to this feature since most of the computed profiles could be classified into two distinct groups : either they varied quickly with variations of the upper boundary assumption or they appeared to change very slowly through a wide (though not unlimited) range of values of ϕ.

Another feature of the plotted curves is that the compact groups such as the short-wavelength group of Fig. 1 usually portray waves whose amplitude factors attain only one or two maxima in the troposphere and, apart from a few exceptions, these waves tend to become small near the tropopause. The broad groups on the other hand are usually associated with amplitude curves which vary much more rapidly with height especially in the upper troposphere.

An example of this feature is illustrated in Fig. 2 which shows two compact groups of curves for wavelengths of 4·5 and 8·9 km and a broad group comprising a set of longer computed wavelengths. The two short waves are confined to the troposphere and it is encouraging to note that Corby detected a wavelength of 8·9 km for this particular case. The shorter 4·5 km wavelength was not apparent in his study but inspection of the topography upwind of Leuchars suggests that if this short wave did exist then its maximum vertical velocity would have been smaller than that of the 8·9 km wave and may therefore have escaped detection.

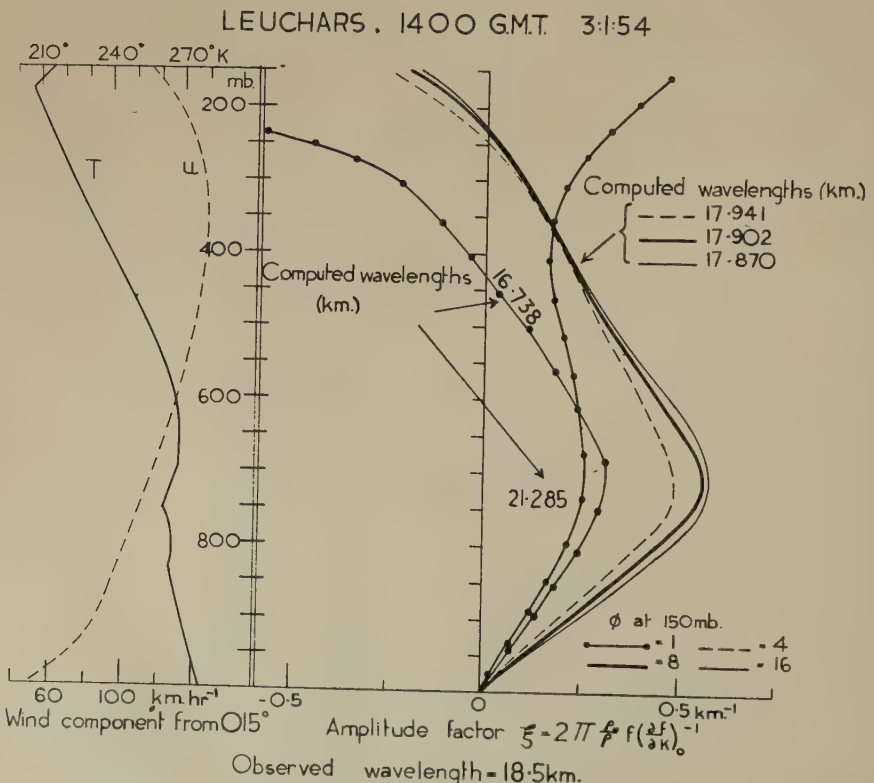

Figure 3 (see Fig. 1 for explanation).

The presence of the broad group of computed longer waves also requires some explanation, for there is no evidence of their actual existence at Leuchars at 04 GMT on 0 November 1953. Since the computed value of K and the amplitude of the calculated waves differ greatly according to the choice of upper-boundary conditions, there is no reason to think that these solutions correspond to any real dynamical property of the airstream. If any waves do exist in this band of wavelengths, they are inadequately treated by this method, either because they depend on conditions at higher levels or because of inaccuracies in the method.

Some computing tests have shown that truncation errors may be appreciable for the amplitude curves which exhibit large values of $\partial^2\xi/\partial P^2$ at high levels, but it is not yet possible to make a general statement on the effect of these errors. Until more exhaustive tests have been carried out we must be wary of attaching undue significance to curves of this type.

Thus the compact groups of ξ curves may be subdivided into two categories according to whether they have large or small values of $\partial^2\xi/\partial P^2$ at high levels. The significance of the first category is doubtful but the compact groups of curves without large curvature at high levels may legitimately be interpreted as real waves whose wavelength and amplitude are almost independent of conditions in the high stratosphere. Of the 19 cases examined 13 yielded curves which could be included in this latter category. The observed and computed wavelengths for these cases are listed below.

TABLE 1. OBSERVED AND COMPUTED LEE WAVELENGTHS

Date	4.11.53	18.11.53	20.11.53	21.12.53	3.1.54	3.1.54	4.1.54
Time (GMT)	1400	1400	0400	1400	0300	1400	0200
Observed wavelength (km)	13·3	13·3	8·9	10·0	18·5	18·5	10·0
Computed wavelength (km)	14·6	15·0	8·9 and 4·5	13·0	17·0	17·9	10·0
Date	11.1.54	12.1.53	23.1.54	11.4.54	15.4.54	15.4.54	
Time (GMT)	1400	1600	1400	1400	0200	1400	
Observed wavelength (km)	5·9	8·9	5·6	8·7	18·7	14·4	
Computed wavelength (km)	7·6	11·0	5·7	7·8	23·4	11·5	

Two of the remaining six cases produced compact groups of curves with large values of $\partial^2\xi/\partial P^2$ at high levels while the four other cases revealed no definitely compact groups. The nature of the curves obtained in these four cases may be illustrated by reference to Fig. 3. On this occasion $\phi = 4$, 8 and 16 led to a compact group of curves with wavelengths close to 17·9 km but $\phi = 1$ failed to produce a curve in this group; instead of one significant curve we obtained two, both of which vary quickly with changes of ϕ. This illustrates the fact that although we may derive the wavelength of about 17·9 km by making only an approximate guess at the appropriate value of ϕ, our guess must at least be within some wide range. In the four unsolved cases it appeared that none of the trial values of ϕ was within an appropriate range; the ξ profiles comprised only chaotic sets of curves which could not be analysed into compact groups. It was impractical in these four cases to extend the search for suitable values of ϕ, especially since the high-level conditions tended to amplify inaccuracies in the method of computation. These inaccuracies are mainly due to the courseness of the finite difference interval of 50 mb at high levels. It is intended to reduce this interval in future numerical work.

As a further guide to the realism of the computed amplitude factors a comparison between the vertical motion anomalies of the radio-sondes and the calculated vertical speeds was attempted. Usually it was difficult to assess the values of H and b applicable to the upwind terrain. Nevertheless, estimates made showed the computed ξ-curves with

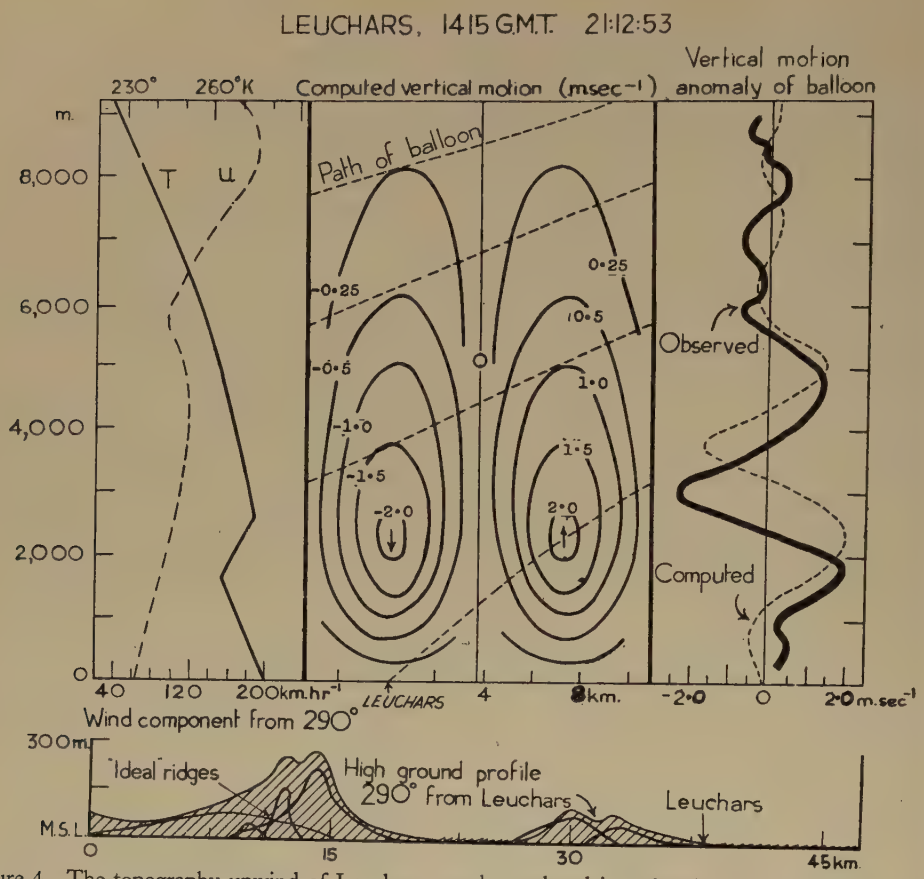

Figure 4. The topography upwind of Leuchars may be analysed into the six 'ideal' ridges shown. The central section of the figure shows the computed vertical motion field over and downwind of Leuchars. The pattern is repeated through each wavelength; the passage of the radiosonde has therefore been adjusted to save space.

their mean maxima of 0·86 km^{-1} at an average level of 780 mb to be reasonably consistent with the general magnitude and distribution of the observed vertical speeds.

In one case it was feasible to analyse the high ground profile in detail sufficiently far upwind of Leuchars to include the most effective topographical features. At 1415 GMT on 21 December 1953 an airstream, whose temperature and wind structure are shown in Fig. 4, blew from about 290° across the Sidlaws to Leuchars. Upwind of Leuchars the high-ground profile could be considered as a synthesis of six 'ideal' ridges as far as the Tay valley. For some considerable distance further upwind the topographical features did not appear suitable for setting off lee waves on this occasion. Fig. 4 shows the computed vertical-motion field in the lee wave over and downwind of Leuchars. Comparison of the observed and computed vertical-motion anomalies of the radio-sonde shows that the computations yield vertical speeds of approximately correct magnitudes. There does appear to be a phase difference between the observed and computed wave but this appears to arise from the two-dimensional treatment of two of the 'ideal' ridges; the 290° airstream crossed these two ridges close to their northeastern ends and the associated lee-wave amplitudes may not therefore have attained their full two-dimensional values.

5. Conclusion

Comparison of computed lee-wavelengths with those estimated by Corby from the rate of ascent of the radiosonde balloons indicates a reasonable accuracy in the computed wavelengths.

The method of calculation as adopted in the present experiment is likely to be inaccurate for lee-waves which have a significant amplitude in the stratosphere. It is not known whether the failure to determine the wavelength satisfactorily in 4 out of the 19 cases is for this reason.

Acknowledgment

We are indebted to the Director-General of the Meteorological Office for permission to publish this paper.

References

Corby, G. A. 1957 *Quart. J. R. Met. Soc.*, **83**, p. 49.
Scorer, R. S. 1949 *Ibid.*, **75**, p. 41.
1953 *Ibid.*, **79**, p. 70.

Appendix

Observed and computed wavelengths

In the table below each line of wavelengths is associated with ξ-curves which have similar characteristics or which fall into compact groups. (Wavelengths are given in kilometres).

Characteristics of ξ-curves: C = compact group, X = accuracy doubtful, n = almost negligible values.

e ae r)	Observed wavelength	Computed wavelengths for various values of				Character of curves
		$\phi = 0$	1	4	16	
	13·3	14·6	14·6	14·6	14·5	C
53		10·6	9·8	9·1	8·7	
		6·9		4·6	2·8	
		$\phi = 0$	1	4	16	
		22·2				
·53	13·3		15·2	15·0	15·0	C
		14·8				
			11·5	5·6	2·6	n
		$\phi = 0$	1	8	16	
		19·1	11·3	47·2	44·5	X
·53	8·9	8·9	8·9	8·9	8·9	C
		4·5	4·5	4·5	4·5	C
				3·7	2·6	n
		$\phi = 0$	1	4	16	
	26·7	20·8	61·5	39·3	37·4	
·53			13·0	6·3	3·0	n
		$\phi = -4$	1	4	8	
		63·4				X
2·53	10·0	13·0	13·0	13·0	13·0	C
			10·2	6·1	4·3	n
		$\phi = 0$	1	4	16	
			17·4			X
4	18·5	16·9		17·0	17·0	C
			14·6			X
				5·9	2·6	n
		$\phi = 1$	4	8	16	
		21·3				
4	18·5		18·0	17·8	17·8	C
		16·7				
					2·8	n
		$\phi = 0$	1	8	16	
		22·0	18·7	14·6	14·1	X
4	10·0	10·0	10·0	10·0	10·0	C
		6·5	5·7	3·5	2·6	n
		$\phi = 0$	1	8	16	
		16·5	15·6	15·0	15·0	CX
54	12·0	10·6	10·0	9·8	9·6	CX
		9·1	8·9	8·9	8·9	CX
		7·2	5·4	2·4		n
		$\phi = 0$	1	4		
		12·0	12·0	12·0		CX
54	5·9	7·6	7·6	7·6		C

Date Time (GMT)	Observed wavelength	Computed wavelengths for various values of				Character of curves
		$\phi = -16$	0	16		
1600		18·7	30·9	20·2		X
12.1.54	8·9	10·9	11·3	10·9		C
			8·5	2·6		
		$\phi = 0$	1	8	32	
0200		32·4	21·3	18·7	17·8	X
14.1.54	10·4	16·1	14·8	13·3	13·3	X
		10·0	8·2			n
				3·5	1·9	n
		$\phi = 0$	1	4	16	
1400		20·4	20·6	20·6	20·4	CX
23.1.54		8·3	8·3	8·3	8·3	CX
		7·4	7·4	7·4	7·4	CX
	5·6	5·7	5·7	5·7	5·7	C
		2·4	2·4	2·4	2·2	n
		$\phi = 0$	1	4	16	
1400		11·8	11·8	11·8	11·8	CX
24.1.54		9·1	9·1	9·1	9·1	CX
	4·8	5·7	5·7	5·7	5·7	n
		3·5	3·3	2·8	1·9	n
		$\phi = 0$	1	8	32	
0200		37·8	26·1	21·3	21·3	X
21.3.54	8·7	18·3	17·2	14·1	13·2	
		7·4	6·7	3·9	2·0	n
		$\phi = 0$	1	8	16	
1400				34·6	29·1	
11.4.54		11·3	10·9	10·2	10·2	CX
	8·7	7·8	7·8	7·8	7·8	C
		6·9	6·1	3·5	2·6	n
		$\phi = 0$	1	4	16	
1500			78·4			X
14.4.54	24·1	22·2		24·1	23·5	
			20·4			
				7·2	3·0	n
		$\phi = 1$	8	16	32	
0200		26·3				
15.4.54	18·7		23·7	23·3	23·3	C
		12·4				
			4·8	3·3	2·4	n
		$\phi = 0$	4	16		
1400		18·5				n
15.4.54	14·4	11·5	11·5	11·5		C
			6·5	3·1		n

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Study of the SO_2 oxidation in solution and its role in atmospheric chemistry

By Chr. E. JUNGE and T. G. RYAN
Air Force Cambridge Research Center, Bedford, Mass., U.S.A.

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Summary

The mechanism of SO_2 oxidation in diluted catalytic solutions was studied. It was found that the oxidation stops after a certain pH value is reached. In the same catalytic solutions, the maximum possible SO_4^{--} formation is a linear function of the SO_2 partial pressure in the air.

This mechanism of SO_4^{--} formation can account for observed SO_4^{--} concentrations in rain, fog, and smog only if some neutralizing cations in the aerosols or traces of NH_3 in the air are present. It is shown that in the latter case the system SO_2-NH_3-catalytic solution can explain the main features of SO_4^{--} formation in our atmosphere.

1. Introduction

It has long been known that sulphate is a very common constituent of rain and fog water (Eriksson 1952, Houghton 1955). In rain collected over the ocean a high percentage of the SO_4^{--} comes from sea water (unpublished data from the rain-water network of the U.S.A. and Atlantic). Approaching the continents, the fraction of SO_4^{--} of other than sea-water origin increases rapidly, owing to both an increase in the absolute amount of this additional SO_4^{--} and a decrease in the sea-salt content. It is likely that most of this SO_4^{--} does not enter the atmosphere as aerosol particles, but is continuously formed by oxidation of SO_2 within the atmosphere itself. For polluted atmospheres with high SO_2 concentrations, this oxidation primarily occurs in cloud and fog droplets (Coste and Courtier 1936). But SO_2 is a very widespread trace gas in our atmosphere (Junge 1956) and it is therefore reasonable to conclude that, for unpolluted air also, oxidation of SO_2 in solution constitutes, besides sea water, the major source of sulphate in rain.

Although the suggestion about the role in air chemistry of SO_2 oxidation in droplets, especially in smog formation, is by no means new, almost nothing is known about its chemistry. The chemical literature contains no quantitative information on the oxidation of SO_2 to SO_4^{--} in water or dilute solutions, and even the qualitative statements are rather inconsistent and unclear (Mellor 1947). In this study, therefore, an attempt was made to obtain enough information on this problem for application to air chemistry without going into the details of the chemical reactions themselves.

2. Experimental investigation of the SO_2 oxidation in dilute solution

(a) *Experimental arrangement*

For the study of the SO_4^{--} formation in water, a very dilute SO_2 in dry air mixture was bubbled at a constant rate of 10-20 cm^3/sec through 30 cm^3 of reagent solution. The dilute SO_2 atmosphere was produced by passing air over a very fine capillary from which SO_2 streamed forth under a pressure of up to 300 mm Hg. The flux of SO_2 was regulated by this pressure on which it depended in a linear way. The bubble tube was specially constructed to provide a maximum contact between gas and solution. The concentration of the SO_2 in the air stream was determined as SO_4^{--} after absorbing and oxidizing in a very dilute H_2O_2 solution the total amount of SO_2 contained in a known volume of air. The air used was dry tank air, cleaned by a long glass-wool filter and by a freezing trap at $-70°C$. The SO_2 was commercially supplied in a small tank. In order to minimize the evaporation of the small solution of 30 cm^3 in runs which lasted for hours, the air was humidified after cleaning and before the addition of SO_2. To keep the volume of the solution constant, any loss by evaporation was compensated by adding distilled water at adequate intervals.

After the exposure of the reagent solution to the SO_2 atmosphere, it was flushed 20-30 min with clean air to eliminate the unoxidized, truly dissolved SO_2. The subsequent SO_4^{--} analysis was done by the Ba Cl_2 nepholometric method of Keily and Rogers (1955). In contrast to other Ba Cl_2 methods, the Ba Cl_2 is stabilized, besides other things, by adding alcohol so that the values are very reliable and reproducible within 5-10 per cent down to SO_4 concentrations of less than 1 γ/cm^3 (1 $\gamma = 10^{-6}$ g).

(*b*) *Results*

A considerable number of runs were made before the essential features of the SO_2 oxidation were properly understood. It appeared that oxidation does not take place in the absence of a catalyst and is not photosensitive with normal daylight. After this preliminary work, the quantitative relationships of the various parameters were established in final runs, on which we will subsequently report.

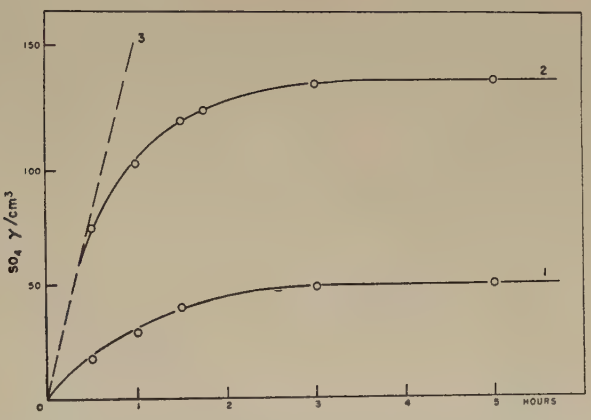

Figure 1. SO_4^{--} concentration as a function of time in a solution of 1 γ/cm^3 Fe Cl_2. Curve 1 : SO_2 concentration $1{\cdot}5 \times 10^4$ γ/m^3; Curve 2 : SO_2 concentration $4{\cdot}2 \times 10^4$ γ/m^3; Curve 3 : calculated SO_4^{--} formation in the solution, if all SO_2 would be oxidized under the conditions of Curve 2. For SO_2, 1 $\gamma/m^3 = 3{\cdot}51 \times 10^{-4}$ parts per million (ppm) by volume.

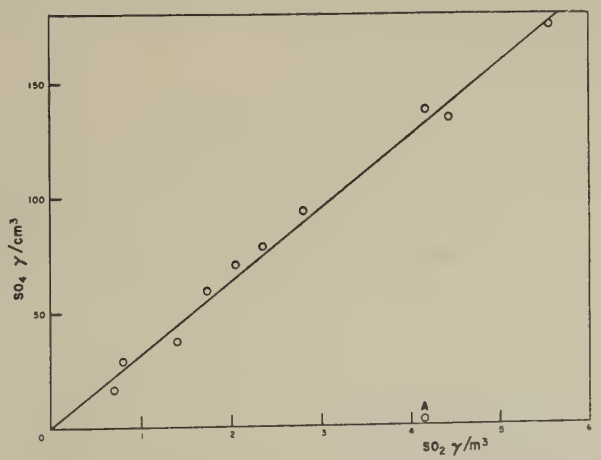

Figure 2. Relationship between the final SO_4^{--} concentration in a solution of 1 γ/cm^3 Fe Cl_2 and the SO_2 concentration. Point A marks a run without any catalyst.

It soon became apparent that the SO_4^{--} formation stops after reaching a certain value (Fig. 1). These and all subsequent runs, unless stated otherwise, were made in solutions containing 1 γ/cm^3 $FeCl_2$ which acted as catalyst. The final value of SO_4^{--} concentration is reached after three hours in our arrangement. The H_2SO_4 formation by oxidation in aqueous solutions is an irreversible process, as is well known and was again carefully checked by evaporating the solutions in order to see if the amount of SO_4^{--} would decrease. Because of this fact, the curves in Fig. 1 do not indicate a reaction equilibrium but rather a negative self-catalytic reaction. As we shall see later, the SO_2 oxidation is controlled by the pH value of the solution.

It is immediately apparent that this final SO_4^{--} content in the solution, reached in our arrangement after 3 hr, is of special interest and it is this value with which we deal throughout the paper.

Fig. 2 shows that this final SO_4^{--} value is a linear function of the SO_2 concentration within the range investigated. Point A shows that practically no SO_4^{--} is formed in the absence of any catalyst.

TABLE 1. THE EFFECT OF CONCENTRATION OF $FeCl_3$ SOLUTION ON SO_4^{--} FORMATION (3 hr VALUES). SO_2 CONCENTRATION $4{\cdot}15 \times 10^4$ γ/m^3

$FeCl_3$ concentration (γ/cm^3)	SO_4^{--} concentration (γ/cm^3)
10^{-3}	9
10^{-2}	50
10^{-1}	100
1	138
10	300
10^2	445

Table 1 indicates the variation of the SO_4^{--} formation over a range of four orders of magnitude of concentration of $FeCl_2$. Compared to the increase of $FeCl_2$, the increase in SO_4^{--} formation is small.

Table 2 compares various substances in their efficiency to act as a catalyst. Most of them were known from literature (e.g., Mellor 1947) to be active. No attempt was made, however, to verify whether these substances acted in the strict sense of catalysts, or to obtain a complete list of compounds. The data shall merely give an impression of the range of variation which is encountered.

TABLE 2. THE EFFECT OF VARIOUS SOLUTIONS ON SULPHATE FORMATION. CONCENTRATION OF THE SOLUTIONS 1 γ/cm^3, SO_2 CONCENTRATION 51×10^4 γ/m^3. SO_4^{--} CONCENTRATIONS AFTER 3 hr

Solution of	SO_4^{--} concentration (γ/cm^3)
$MnCl_2$	329
$CuCl_2$	199
$FeCl_2$	167
$CoCl_2$	49
NH_4OH	49
NaCl	4
dist. H_2O	3

Variation in the illumination from sunlight down to complete darkness had no effect n the SO_4^{--} values. Since the test solution was enclosed in glass, restricting the trans- ission of short-wave radiation, the possibility is not excluded that at shorter wave ength a photochemical influence exists. All runs were made with the same illumination.

Fig. 3 shows the influence of the pH value of the solution on the SO_4^{--} formation. he pH value of a 1 γ/cm^3 $FeCl_2$ solution was varied by adding hydrochloric acid or itric acid. With the SO_2 concentration used in these runs, a pure $FeCl_2$ solution of γ/cm^3 which had a pH of 5·5 finally reached a pH of 2·2 as a result of H_2SO_4 formation. he curves indicate that approximately only so much H_2SO_4 is formed as to change ne original pH of the hydrochloric or citric acid solution to 2·2, so that SO_4^{--} formation eyond pH = 2·2 is almost stopped. The two curves differ a little from each other and o not agree completely with the expected SO_4^{--} values. This may be due to some catalytic ffect of the added acids themselves. But the data leave no doubt that the major factor hich restricts the SO_4^{--} formation is the hydrogen-ion concentration.

As in indirect proof that the pH of the solution is the important factor in stopping he oxidation, the following experiment was performed : an air stream was bubbled hrough a dilute NH_4OH solution, in order to obtain a small but constant NH_3 atmos- here, and combined with the SO_2 stream just before entering the bubble tube for SO_4 ormation. The bubbling of this combined gas stream, which had a concentration of 5,500 γ/m^3 NH_3 and 31,000 γ/m^3 SO_2, was continued for 30 hr. The analysis of the olution after the run indicated that 77 per cent of the total available SO_2 was absorbed n the solution, which had a final pH of 2·47. This result shows that by keeping the ydrogen-ion concentration low, the oxidation process is unrestricted.

There is, of course, no reason why the results obtained on a volume of 30 cm^3 cannot e readily applied to the much smaller volumes of fog, cloud, or rain droplets. The only ifference is the time involved for the completion of the reaction. In our experiment it ook not more than three hours to reach the final SO_4^{--} concentration. The curves in ig. 1 seem to indicate that, at the beginning, the supply of SO_2 is the rate-controlling actor; but later, upon approaching the final SO_4^{--} level, the oxidation process itself s decisive for the reaction rate. To obtain further information, droplets of 2 and 4 mm iameter of a 1 γ/cm^3 $FeCl_2$ solution and of pure water containing various colour indica- ors for pH were exposed to a stream of our dilute SO_2 atmospheres. The expected inal pH value, according to the SO_2 concentration, was 2·0. The droplets assumed a H of about 3·5 after a few minutes. When flushed with pure air, the droplets containing

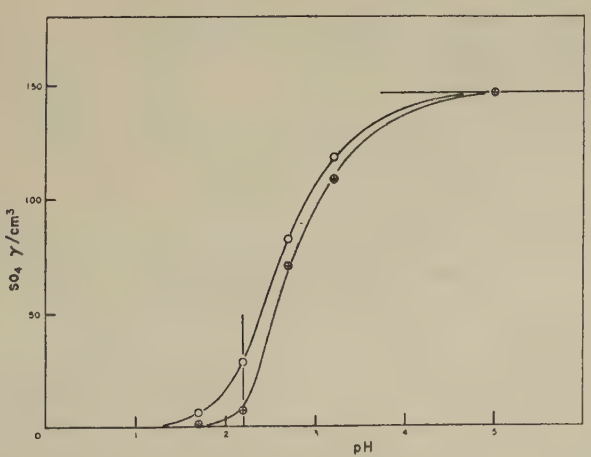

Figure 3. Formation of SO_4^{--} in a 1 γ/m^3 $FeCl_2$ solution of various pH values.
Open circles : pH adjusted by addition of hydrochloric acid.
Crossed circles : pH adjusted by addition of citric acid.

no catalyst returned rapidly to neutral, whereas the droplets containing the $FeCl_2$ maintained their pH, as was expected. But even after the maximum possible exposure time of 10-15 min (because of evaporation) the pH did not decrease very much beyond 3·5 indicating that in this size range of volumes the completion of oxidation may take an hour or so as against the much faster diffusion process.

We made another experiment : an NaCl spray aerosol was produced in a polyethylene bag of 0·7 m^3 volume. The humidity was kept above 70 per cent so that the particles were droplets. Known amounts of SO_2 and NH_3 were added. Samples of this aerosol were obtained by an impactor and analyzed microchemically. Since the microchemical methods for SO_4^{--} were not sensitive enough, we used the formation of $(NH_4)_2SO_4$ in the aerosol as an indication of the SO_4^{--} formation by analyzing for NH_4^+ a valid technique according to the 30-hr run discussed. Since the concentration of the spray aerosol decreased with time due to fallout, the *ratio* of NH_4^+ to Cl^- in the aerosol served as a measure of the amount of SO_4 formed.

After the bag was filled with spray, NH_3 was added to obtain a concentration of about 1,000 γ/m^3. Following this, the aerosol showed no NH_4^+ content. After addition of SO_2 of similar concentrations, however, the NH_4 content of the aerosols increased rapidly until after 15 min a constant NH_4^+/Cl^- ratio was reached. Measurements indicated that after this time there were still considerable amounts of the gas traces left. This confirms our previous result that only a certain amount of SO_2 is oxidized and thereafter the reaction ceases. Because of the presence of NH_3, a much larger amount of SO_4^{--} had to be formed. In spite of this, however, the time for completion of the process was much shorter than in the runs with the bubble tube, 15 min against 3 h (and more in the presence of NH_3), indicating that the rate of oxidation may decrease with the ratio of volume to surface of the solution. The aerosol particles had an average size of about 0·5 μ radius.

Actually, in this experiment we already dealt with the more complicated SO_2-NH_3-liquid water system which will be treated in greater detail later. Unfortunately, this static method with the polyethylene bag did not allow satisfactory quantitative measurements because of gas absorption and reaction on the walls, fallout of aerosols, etc. and was, therefore, dropped in the course of the investigations.

These experiments leave the question of the time for the completion of the SO_2 oxidation in volumes of the size of fog droplets open within a wide range. In the presence of NH_3 it is longer than 15 min and is probably shorter than a few hours. We estimate that it is about one hour for droplets of 10 μ radius.

We may point out that no attempt was made to investigate the chemistry and kinetics of the pH controlled SO_2 oxidation. It was suggested that the SO_4^{--} formation in solution simply stops because the solubility of SO_2 decreases rapidly with the pH-value, so that from a certain point on, there is no SO_2 present for further oxidation. The fact that for the same SO_2 concentration the final SO_4^{--} value depends on the catalyst, immediately indicates that the process is more complicated. Besides this, direct measurements showed that dissolved SO_2 was still present in the solution after oxidation came to a stop.

3. The system SO_2-NH_3-liquid water and the formation of sulphate in rain and fog

(*a*) *The system* SO_2-NH_3-*liquid water*

In applying these experimental results to air chemistry, one is faced with the fact that very few data on SO_4^{--} formation in the atmosphere are available. Nothing is known, for instance, about the kind and concentration of catalytic material in cloud and fog droplets. Furthermore, no simultaneous measurements are available on the SO_2 and NH_3 concentration in air on the one hand, and the SO_4^{--} concentration and the liquid-water content of the cloud droplets which are formed in this air on the other hand. In the subsequent quantitative discussion of the system SO_2-NH_3-liquid water we can

show that the general features of the SO_4^{--} formation in the atmosphere can well be explained on this basis, but the figures used in the calculations and the agreement with the observations should be understood as orders of magnitude only. Despite this, we feel that such a discussion is justified as a first attempt to bring order in these phenomena and a stimulation for further work in this field.

From Fig. 1 it can be inferred that the final SO_4^{--} concentration can be expressed as a linear function of the SO_2 concentration

$$(SO_4^{--})_s = c\,(SO_2)_g \quad . \quad . \quad . \quad . \quad (1)$$

Here, s denotes the concentration in the solution in γ/cm^3 and g denotes the concentration in the gas phase in γ/m^3. The constant c is dependent on the catalyst and probably on the temperature and the presence of other substances in the solution. If we assume the presence of a catalyst in rain or fog water of the same efficiency as a 1 γ/cm^3 $Fe\,Cl_2$ solution, $c = 3{\cdot}3 \times 10^{-3}$. In clean, country air, remote from industrial centres, $(SO_2)_g$ is about 10 γ/m^3 which would give a $(SO_4^{--})_s$ value of $3{\cdot}3 \times 10^{-2}$ γ/cm^2, a concentration which is about two orders of magnitude below the observed values in rain and three orders of magnitude less than in fog water (Houghton 1955). The system SO_2-liquid H_2O alone is, therefore, of little importance. We know, however, that NH_3 is always present in our atmosphere (Junge 1956) and that we really deal with the system NH_3-SO_2-liquid H_2O.

To treat this system, we first have to give some details on the simpler system, NH_3-liquid H_2O (Egnér 1932). The dissolved NH_3 is present as $NH_4\,OH$, which is partially dissociated according to the mass law with the concentrations in [ ] given in moles per litre :

$$K = [NH_4^+] \times [OH^-] \times [NH_4\,OH]^{-1}$$

The undissociated ' free ' ammonia in solution is in equilibrium with the NH_3 partial pressure, according to Henry's law : $[NH_3]_{air} = \beta\,[NH_3]_{water} = \beta\,[NH_4\,OH]_{water}$. Because of the dissociation of the water, we also have $K_W = [H^+] \times [OH^-]$. For the ratio F of the total concentration of ammonia in air to that in solution we obtain :

$$F = [NH_3]_{air} \times ([NH_4^+] + [NH_4\,OH])^{-1} = \beta \times (1 + K\,[H^+]/K_W)^{-1} \quad . \quad (2)$$

With $[H^+] > 10^{-7}$, that is, for acid solutions of pH < 7, we find approximately

$$F = \beta\,K_W/K\,[H^+] \quad . \quad . \quad . \quad . \quad (3)$$

In the combined system NH_3-SO_2-liquid H_2O the following process goes on. If the liquid H_2O contains a catalyst, the formation of SO_4^{--} starts. In the presence of NH_3 gas, the pH-value, which corresponds to the given SO_4^{--} concentration, can, of course, not be attained until enough NH_3 is absorbed for the proper adjustment of the F-value. The amount of SO_4^{--} neutralized by NH_3 in the solution is then so large compared with the ' free ' SO_4^{--} that approximately the stochiometric ratio according to $(NH_4)_2\,SO_4$ is realized.

For quantitative application to SO_4^{--} formation in fogs and clouds, we calculate the total amount of NH_4^+ and SO_4^{--} in solution in the droplets when the partial pressures of NH_3 and SO_2 in γ/m^3 and a liquid-water content of l g/m^3 are given.

Since for very dilute $H_2\,SO_4$ $[H^+] = 2{\cdot}08 \times 10^{-5}\,(SO_4^{--})_s$, relation (1) can be written

$$[H^+] = c_1\,(SO_2)_g;\ c_1 = 2{\cdot}08 \times 10^{-5}\,c. \quad . \quad . \quad . \quad (4)$$

Combinations of (3) and (4) yield for the total amount of NH_4^+ in solution

$$(NH_4^+)_s = 2{\cdot}2 \times 10^{-11}\,K\,c\,(NH_3)_g\,(SO_2)_g/\beta\,K_W$$

A fog or cloud with liquid-water content l, therefore, contains the total amount of NH_4^+ per m^3 (indicated by ( ) without notation)

$$(NH_4^+) = 2{\cdot}2 \times 10^{-11}\,K\,c\,(NH_3)_g\,(SO_2)_g\,l/\beta\,K_W$$

The corresponding value for (SO_4^{--}) equals 2·67 (NH_4^+) under the assumption that the composition corresponds to $(NH_4)_2\,SO_4$.

The constants have the following values at 10°C :

$$K = 2{\cdot}0 \times 10^{-5} \text{ (mole/litre)}$$

$$\beta = 3{\cdot}6 \times 10^{-4}$$

$$K_W = 3{\cdot}6 \times 10^{-15} \text{ (mole/litre)}.$$

c depends very strongly on the kind and concentration of the catalyst. As a reasonable estimation we can choose a value corresponding to ammonium (see Table 2) at a concentration of 1 γ/cm^3, which gives $c = 10^{-3}$.

Thus we obtain :

$$(SO_4^{--}) = 0{\cdot}87 \times (NH_3)_g\,(SO_2)_g\,l \;. \quad . \quad . \quad . \quad (5)$$

$$(NH_4^+) = 0{\cdot}33 \times (NH_3)_g\,(SO_2)_g\,l \;. \quad . \quad . \quad . \quad (6)$$

$$\text{pH} = +\,6{\cdot}8 - \log\,(SO_2)_g \quad . \quad . \quad . \quad . \quad (7)$$

The $(NH_3)_g$ and $(SO_2)_g$ concentrations in these formulae indicate, of course, the values *after* the completion of the fog formation and the SO_2 oxidation. The *original* gas concentrations, *before* the fog formed, are :

$$(SO_2)_0 = (SO_2)_g + 2/3\,(SO_4^{--}) \quad . \quad . \quad . \quad . \quad (8)$$

$$(NH_3)_0 = (NH_3)_g + 17/18\,(NH_4^+) \;. \quad . \quad . \quad . \quad (9)$$

(*b*) *Application to air chemistry*

With the help of (5), (6), (8) and (9), (SO_4^{--}) and (NH_4^+) can easily be expressed as functions of the original gas concentrations $(SO_2)_0$ and $(NH_3)_0$ and the liquid-water content. To indicate the type of result and the order of magnitude of the sulphate formation, we will consider two numerical examples which represent typical atmospheric conditions of unpolluted and polluted air :

Example 1. For fairly clean country air, as along the north-east coast of the U.S., it can be assumed

$$(SO_2)_0 = 20\ \gamma/m^3$$

and

$$(NH_3)_0 = 3\ \gamma/m^3 \text{ (Junge 1957)}.$$

Example 2. For highly-polluted air it can be assumed

$$(SO_2)_0 = 500\ \gamma/m^3$$

and

$$(NH_3)_0 = 10\ \gamma/m^3.$$

Unfortunately, there are very few NH_3 data available for polluted atmospheres, but 10 γ/m^3 appears to be reasonable. On the basis of these figures the amount of SO_2 oxidized in cloud and fog droplets as a function of the liquid-water content is shown in Fig. 4. The maximum possible amount of (SO_4^{--}) is, of course, reached as $l \to \infty$ and is controlled by the $(NH_3)_0$ value. At a liquid-water content of 0·1 g/m^3, which is a reasonable average value for fogs, some additional data for these two examples are compiled in Table 3.

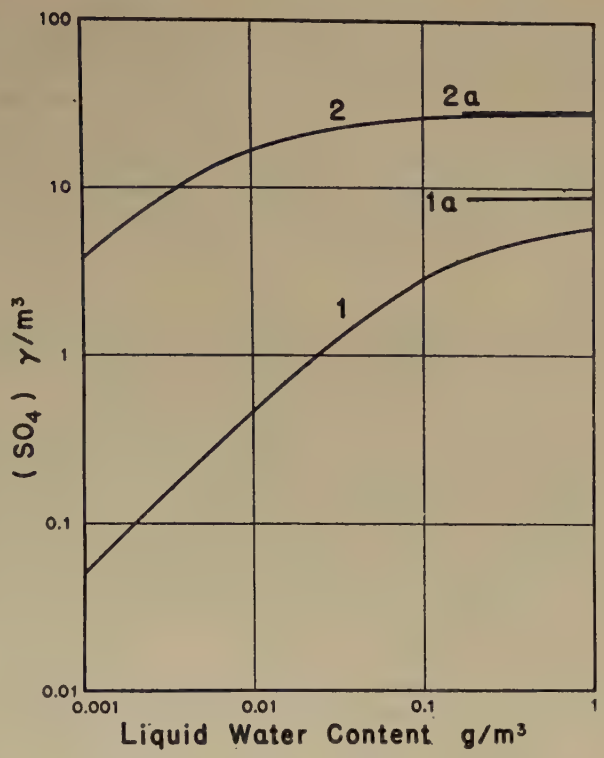

Figure 4. (SO_4^{--}) values as functions of liquid-water content for the two examples of clean country air (1) and polluted air (2). (1a) and (2a) are the maximum values for large liquid-water contents.

TABLE 3. SOME FIGURES ON THE $(NH_4)_2 SO_4$ FORMATION IN A FOG OF LIQUID-WATER CONTENT OF 0·1 g/m³ FOR THE EXAMPLES 1 and 2 (SEE P. 52)

	$(SO_2)_0$ (γ/m³)	% SO_2 oxidized	(SO_4^{--}) (γ/m³)	$(NH_3)_0$ (γ/m³)	% NH_3 converted to $(NH_4)_2 SO_4$	(NH_4^+) (γ/m³)	pH
country air	20	9·7	2·9	3	34	1·1	5·1
polluted air	500	3·5	26·2	10	98	10·4	4·5

To compare these calculations with observations, we may refer to Houghton's (1955) data on fog at the north-east coast of the U.S. in fairly clean country air. He found in fogs of an average liquid-water content of 0·2 g/m³ SO_4^{--} concentrations in the fog water fluctuating between 3 and 50 γ/cm³, with an average of 10 γ/cm³, corresponding to (SO_4^{--}) values fluctuating between 0·6 and 10 with an average of 2 γ/m³. This agrees with the data given in Fig. 4 and Table 3 for country air. It may be mentioned that the major part of the SO_4^{--} in Houghton's data, even in the sea fog directly at the coast, must have come from sources other than sea water because of the high ratio of SO_4^{--}/Cl^- as compared with that of sea water. The pH of these fog waters averaged around 5. Measurements of the conductivity of the fog water indicated the presence of certain cations which may well have been NH_4^+. Unfortunately, no data on NH_3 and NH_4^+ are available, so that further conclusions cannot be drawn.

The observations in polluted atmospheres are also characterized by a complet lack of data on ammonia. Simultaneous data of SO_2 and SO_4^{--} concentrations sho (e.g. Coste and Courtier 1936, for London) that only a few per cent of the SO_2 is oxidize in agreement with Table 3. Katz (1952) found values of more than 20 per cent in smelte areas, but this fraction is likely to be too high because of the analytical method use It would be of great interest to obtain data on ammonia in polluted atmospheres to se what fraction of the SO_4^{--} is present as $(NH_4)_2\,SO_4$. According to our model (Table 3 sulphate smog should be characterized by the fact that almost all NH_3 is bound to th droplets. Field tests could prove whether the release of additional NH_3 would increas the smog formation.

In our calculations we assumed that the SO_4^{--} in the droplets was completel neutralized by NH_4^+. The excess amount of SO_4^{--} which is responsible for the pH adjust ment of the solution is, in fact, very small, amounting to less than 1 per cent of the (SO_4^{--}) values, so that our calculations are a very good approximation.

Because the SO_4^{--} formation at higher liquid-water contents approaches the maximum value which is determined by the amount of NH_3, the values in Fig. 4 are influenced b the type and concentration of the catalyst *only* for small liquid-water contents. Applie to smog it means that the most important factor in sulphate droplet formation is actuall the supply of NH_3. The steady increase of sulphate smog aerosols, often observed over period of days, may primarily reflect the amount of NH_3 released in that area. It ma be stressed, however, that other cations may augment the action of NH_3, and Ca^{++} which seems to be always present in aerosols, is very likely one of them.

A difficulty in the pH controlled SO_4^{--} formation is the interference at very low SO_2 concentration with the CO_2 in the air. CO_2 is soluble in water, forming undissociate CO_2 as well as CO_3^{--} and HCO_3^- ions. At the partial pressures of CO_2 in air, water in equilibrium with air contains about 90 per cent of the dissolved CO_2 undissociated and 10 per cent as HCO_3^-; whereas, the CO_3^{--} concentration is negligible. The pH is close to 5·6, depending to some extent on the temperature. Water in contact with air readily assumes this equilibrium (Kendall 1916) and, accordingly, no SO_4^{--} formation should occur at SO_2 concentrations less than a certain threshold value (in our example 15 γ/m^3), depending on the type of catalyst. There might be, therefore, a certain low SO_2 concentration in the air which cannot be removed by cloud, fog, or rain, and which should be found, even in very clear air, as a fairly constant minimum value over wide parts of the world. Indications of this are given by observations (Junge 1956, 1957).

If cloud or fog droplets evaporate, the amount of $(NH_4)_2\,SO_4$ which they contain will not change, but the solutions become more concentrated than could be expected from the $(NH_3)_g$ and $(SO_2)_g$ values. Therefore, care must be taken if conclusions are drawn from the SO_4^{--} and NH_4^+ concentration in fog water. The (NH_4^+) and (SO_4^{--}) values, however, do not change in an evaporating cloud, unless the air is mixed with air outside the cloud or fog. Because the majority of cloud drops in the atmosphere evaporate rather than fall as rain, a considerable fraction of the condensation nuclei should contain $(NH_4)_2\,SO_4$, as is actually found on a world-wide basis (Junge 1956). Owing to mixing with the surrounding cloud-free air, the expected (SO_4^{--}) and (NH_4^+) concentrations in the natural cloud-free air should be much lower than the corresponding values for clouds and fogs as, e.g., calculated for country air in Table 3. This is actually the case, as shown by the data of Junge. A condensation nucleus can, of course, run through the cycle of condensation and evaporation several times, with the possible step-wise accumulation of $(NH_4)_2\,SO_4$ in each cycle, a process which may be important in smog or stratus clouds.

Application of the SO_2-NH_3-water system to the sulphate content in rain water is complicated by the fact that a considerable amount of mineral constituents is collected by the raindrops during their fall. Curve 1, Fig. 4 indicates a SO_4^{--} concentration of 6 γ/m^3 for a raining cloud of l g/m^3 liquid-water content in clean country air. Since the level of SO_4^{--} concentration in rain water fluctuates normally between 1 and 3 γ/cm^3 in unpolluted areas, the process can easily explain the observations. However, recent

measurements seem to indicate that in rain water the neutralizing effect of Ca^{++} and Mg^{++} is just as important as that of NH_4^+. Larson and Hettick (1956) summed the SO_4^{--} and HCO_3^- concentrations and found a fairly good correlation with the Ca^{++} plus Mg^{++} concentration. This can be interpreted as a reaction of H_2SO_4 with calcitic or dolomitic dust particles. They found a similar correlation between the sum of $NO_3^- + NH_4^+$-nitrogen and SO_4^{--}. This may imply a correlation between NH_4^+ and SO_4^{--}, but since NO_3^- and NH_4^+ nitrogen are not separated, no definite interpretation with respect to our process can be given. As mentioned above, the pick-up of dust from the raindrops on their way down will further obscure any composition of the original cloud water.

It must be stressed that SO_2 can also be oxidized directly into SO_3 (with subsequent formation of H_2SO_4 if moisture is present) by a photochemical reaction. This process was recently very carefully studied (Gerhard 1954) with the result that this reaction is now known to be of the first order with respect to SO_2 in an excess of O_2, and that the reaction rate is about 0·1 to 0·2 per cent conversion of SO_2 per hour in intense natural sunlight. Although this rate is slow, a few per cent of SO_4^{--} may be found in an air mass in the course of a few days. This process is, of course, not dependent on the presence of of fog droplets and NH_3. No general conclusion is possible at the moment as to the relative importance of these two processes in the sulphur cycle in our atmosphere. While the photochemical formation of SO_4^{--} may be important in certain parts of our atmosphere, the observations in fogs and smogs point to SO_2 oxidation in droplets.

4. Conclusions

In this paper, a first attempt to study in detail the mechanism of gas reactions with aerosols has been described. The particular case of SO_2 and NH_3 investigated is believed to be of importance to air chemistry and is able to explain the major features of the SO_4^{--} content in atmospheric condensation products. It appears that the presence of NH_3 is of considerable importance, but the lack of simultaneous data on this constituent in the atmosphere makes it impossible, for the time being, to check the quantitative conclusions more carefully.

It is unlikely that the proposed mechanism of SO_4^{--} formation is the only one in the atmosphere. We know that SO_2 can be oxidized photochemically by sunlight without the presence of droplets and NH_3, and there might occur reactions with other gases or with aerosol particles containing $CaCO_3$ or other compounds of alkalis or alkaline earth metals. However, it is felt that SO_2 oxidation in droplets is the most important process, especially in polluted atmospheres. If this is so, dense aerosol formations in SO_2 polluted atmospheres may be considerably decreased, not only by reducing the release of SO_2, but also, at least as efficiently, by the judicious decrease of NH_3 and good catalytic aerosols.

References

Coste, J. H. and Courtier, G. B.	1936	*Trans. Faraday Soc.*, **328**, p. 1198.
Egnér, H.	1932	*Med. Centr. försök. jordbruk.*, Stockholm, No. 409, p. 257.
Eriksson, E.	1952	*Tellus*, **4**, pp. 215 and 280.
Gerhard, E. R.	1954	*Dissertation Abst. Ann Arbor*, Michigan, **14**, p. 321. (Univ. Microfilms, No. 6949).
Houghton, H.	1955	*J. Met.*, **12**, p. 355.
Junge, Chr. E.	1956	*Tellus*, **8**, p. 127.
	1957	To be published in *Advances in Geophysics*, **4**.
Katz, M.	1952	*Air Pollution*, New York (McGraw-Hill), p. 580.
Keily, H. J. and Rogers, L. B.	1955	*Anal. Chem.*, **27**, p. 759.
Kendall, J.	1916	*J. Amer. Chem. Soc.*, **38**, p. 1480.
Larson, T. E. and Hettick, I.	1956	*Tellus*, **8**, p. 191.
Mellor, J. W.	1947	*A comprehensive treatise on inorganic and theoretical chemistry*, **10**, London (Longmans Green).

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The empirical relation between solar radiation and hours of bright sunshine in the high-altitude tropics

By J. GLOVER and J. S. G. McCULLOCH
East African Agriculture and Forestry Research Organization, Muguga, Kenya.

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Summary

The relationship between total solar radiation on a horizontal surface (Q), as measured by a pyranometer, and hours of bright sunshine (n), as measured by a Campbell-Stokes recorder, has been examined and it is shown that there is little practical gain in weighting the hourly sunshine records by the sine of the apparent solar altitude.

In the non-dimensional equation

$$Q/Q_A = a + b\,n/N$$

values of a and b derived from daily records are shown to vary widely from month to month but their distribution appears to be substantially normal. Actual mean values of a and b, for a five-year period (1938-1943) at Kabete, Kenya (altitude 6,000 ft) differed slightly but non-significantly according to the length of period in which the data were grouped, the slope of the equation increasing with increased smoothing.

The sum of a and b has been shown to be approximately constant ($\simeq 0{\cdot}82$) at Kabete, where the optical air mass varies little throughout the year. For checking the consistency of the data from various parts of the world, it is essential to reduce the differing apparent transmission coefficients to standard measure. In practice, a simple correction to optical air-mass unity has proved adequate for the records available at this station.

1. Introduction

With the expansion of studies of climate, plant, soil, water relationships in East Africa it was found essential to obtain some estimate of solar radiation, however approximate. Radiation-measuring instruments of the thermopile type are rare and estimates must be derived from simpler devices such as sunshine recorders. Many workers have used the relationship between radiation as measured by a thermopile and sunshine hours as measured by a Campbell-Stokes recorder. In general such a relationship is expressed in the form,

$$Q/Q_A = a + b\,n/N \qquad (1)$$

where Q is the radiation on a horizontal surface at or near ground level, Q_A is the radiation on a horizontal surface at the upper limit of the earth's atmosphere, a and b are regression constants and n/N is the ratio of actual to possible hours of sunshine. Penman (1948) suggests that a seasonal variation may occur in values of a and b and Black, Bonython and Prescott (1954) indicate that there may also be a dependence on latitude. However, Penman (1955) remarks that the relationship which he found suitable for English conditions was also reasonably adequate for Ghana and that no significant latitude effect was observable. In the face of such conflicting evidence it seemed essential to examine the relationship in our local region.

2. Records and instruments

Five years (1938-43) of daily records of solar radiation on a horizontal surface and of hours of sunshine were available from the Meteorological Observatory at Kabete in Kenya. This station was maintained by the East African Meteorological Department at 1°16′S, 36°45′E and almost 6,000 ft above sea level. Its radiation regime was that of the dry, high-altitude tropics; the average daily total of solar radiation on a horizontal surface was 507 g cal cm^{-2} in a 12 hr day over the five-year period under review.

The instrument used is said to have been a Hartmann and Braun pyranometer with a Thirl Multiple Recorder (E. African Met. Dept. 1957). Photographs of similar equipment are given by Riemerschmid (1940). Because of its higher heat capacity it had a somewhat slower response to

change in radiation intensity than a Gorczynski solarimeter in use at The East African Agriculture and Forestry Research Organization, Muguga, some 10 mi from Kabete and some 800 ft higher. A recent comparison of the instruments shows that both give similar results on a bright clear day, but differ slightly under conditions of intermittent cloud. In addition, records obtained from the pyranometer on bright clear days in 1957 were similar to those obtained when it was first installed, so it is unlikely to have departed widely from its original calibration during the period in which it was operating.

The sunshine records were obtained from a tropical model of the Campbell-Stokes recorder. The burn recorded would be expected to be related more closely to radiation on a surface perpendicular to the solar beam than to that on a horizontal surface. As no records of radiation on a surface perpendicular to the solar beam at Kabete were available the direct test of the assumption was impossible. However, it was possible to test the relationships between radiation on a horizontal surface hour by hour and the corresponding weighted hours and unweighted hours of bright sunshine.

3. Solar radiation and weighted hours of sunshine

The weighted hours of bright sunshine were obtained from the actual hours by multiplying each hourly value by the sine of the mean angle of apparent altitude of the sun. This, of course, gives only an approximate weight, but further refinements were considered unnecessary. The results are shown in Fig. 1 : for a clear day in (a) and for partly cloudy and dull days in (b) and (c). It is apparent that radiation hour by hour on a horizontal surface is in general more intimately related to the weighted hours of sunshine than to the actual records. This effect is most marked on bright days, is of lesser extent on partly cloudy days and almost vanishes on dull days.

It would therefore seem that the daily totals derived from weighted sunshine hours would give a better estimate of daily total radiation than that derived from the actual unweighted records.

This was tested by weighting hourly values of bright sunshine over one year (1940) and regressing hourly radiation (Q_h) on weighted (S_w) and on unweighted (S) records. A comparison of the accountable variation (r^2) derived from each regression is given in Table 1.

The slight gain due to the use of weighted values of bright sunshine did not outweigh the labour involved in calculating them. There are two probable reasons for this apparently contradictory result. The first is that partly cloudy days are most frequent and completely bright or dull days are rare; the second is that the Campbell-Stokes instrument with its wide acceptance over-estimates the radiation during the morning and evening hours and under-estimates it during the midday hours. In the end, these extravagant estimations cancel out under East African conditions and a reasonable picture of daily totals of radiation can be obtained from the actual records of the Campbell-Stokes instrument without weighting. Hence, throughout the remainder of this paper, only the actual hours of bright sunshine are considered.

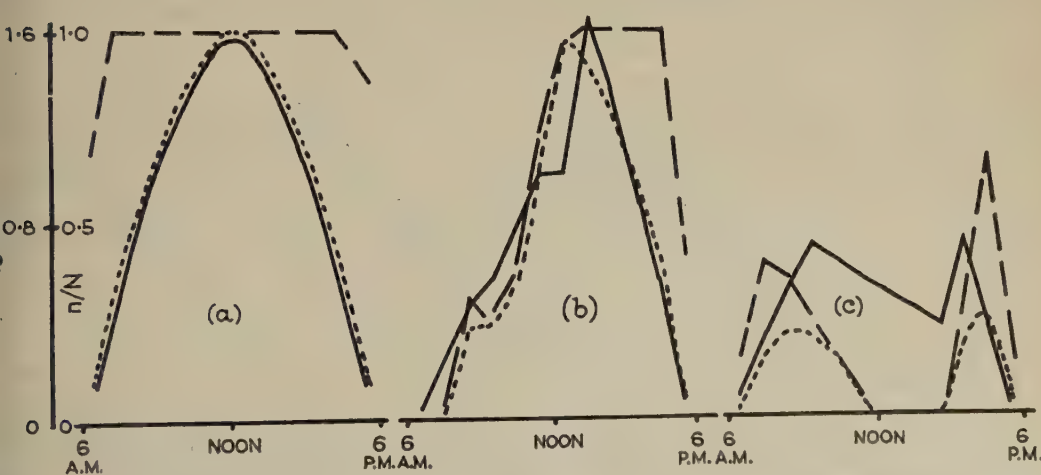

Figure 1. The relationship between radiation (———), actual n/N (— — —) and weighted n/N (- - - - -) on bright days (a), partly cloudy days (b) and dull days (c).

TABLE 1.

Month	r^2 from the regressions Q_h on S_w	Q_h on S
Jan.	0·92	0·91
Feb.	0·92	0·82
Mar.	0·82	0·83
Apr.	0·84	0·82
May	0·81	0·77
June	0·90	0·87
July	0·88	0·85
Aug.	0·92	0·93
Sept.	0·83	0.81
Oct.	0·62	0·65
Nov.	0·89	0·89
Dec.	0·89	0·81

Fig. 2 (a), (b) and (c) show respectively the daily (1938-39), the ten-day mean (1938-43) and the monthly mean (1938-43) values of n/N plotted against the corresponding values of Q/Q_A. There is no marked curvature as is described by Hamon, Weiss and Wilson (1954), although the zero limitation of n/N does produce a non-significant curvature at low levels of radiation. As a result the linear regression of radiation on hours of bright sunshine seems a suitable means of expressing the relationship. The regression lines are shown superimposed on the data.

4. Analysis of records

(a) *The daily records*

The daily records were used to provide expressions for each month, each year and for the whole five-year period. Those for the months are too numerous to publish though they are, of course, available at The East African Agriculture and Forestry Research Organization, Muguga. They show that a and b can vary widely with no obvious relation to the slight seasonal variations at the latitude of this station, and that the extreme variations are associated mainly with unusually large or small amounts of cloud which occur by chance in any particular month. Indeed the distributions of a and of b as tested by Fisher's (1948) method of moments, are substantially *normal*, with mean values of $0{\cdot}27_4 \pm 0{\cdot}02_3$ and of $0{\cdot}54_4 \pm 0{\cdot}02_8$ respectively. Thus a relationship of the form

$$Q/Q_A = 0{\cdot}27 + 0{\cdot}54\, n/N \quad . \quad . \quad . \quad . \quad (2)$$

is a suitable one for local conditions. This is very similar to the corresponding relationships derived for each year and for the whole five-year period, which are shown in Table 2 and Eq. (3) respectively. Hence grouping of the data by months and years has had little effect.

$$Q/Q_A = 0{\cdot}26 + 0{\cdot}57\, n/N, \qquad r^2 = 0{\cdot}85 \quad . \quad . \quad . \quad (3)$$

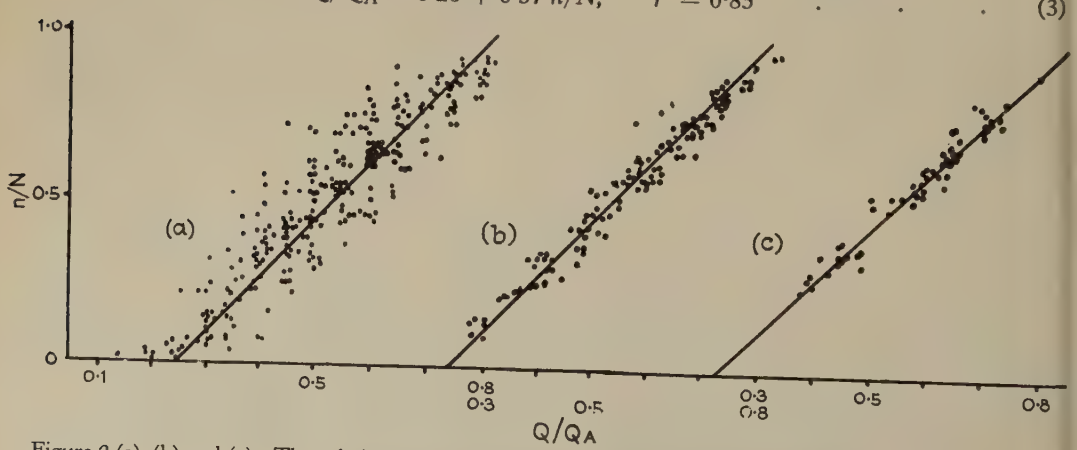

Figure 2 (a), (b) and (c). The relationship between Q/Q_A and n/N for daily, ten-day and monthly records.

TABLE 2. YEARLY VALUES OF a AND b DERIVED FROM DAILY RECORDS

	a	b	r^2
1938	0·24	0·55	0·83
1939	0·27	0·55	0·81
1940	0·29	0·54	0·87
1941	0·26	0·58	0·87
1942	0·24	0·59	0·87

Further study of the monthly records indicated that the sum of a and b is remarkably constant with a mean value of 0·82. This clearly suggested an inverse relationship between a and b, which was in fact given by the equations

$$b = 0{\cdot}83 - 1{\cdot}06\,a \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$$a = 0{\cdot}66 - 0{\cdot}71\,b \quad . \quad . \quad . \quad . \quad . \quad (5)$$

with a highly significant correlation coefficient of 0·85. Within the limits of the data these regressions are almost indistinguishable and for convenience it may be assumed that the sum of a and b is 0·82. This is clearly a measure of the apparent transmission coefficient obtained by substituting unity for n/N in Eq. (1) and indeed actual records on clear days show values from 0·78 to 0·86. The apparent transmission coefficient will be written t^m where m is the optical air mass at sea level. Clearly for high-altitude stations the value of m must be corrected by a factor p/p_0, the ratio of the atmospheric pressure at the station to that at sea level. In this paper, this correction is ignored for stations whose altitude is less than 500 ft.

(*b*) *The ten-day means*

The ten-day mean relationship, derived in a similar manner, is, for the five-year period

$$Q/Q_A = 0{\cdot}24 + 0{\cdot}59\,n/N, \qquad r^2 = 0{\cdot}90 \quad . \quad . \quad . \quad (6)$$

The ten-day means when cross-totalled, that is the mean of all the first ten-day periods in January and so on, gave

$$Q/Q_A = 0{\cdot}23 + 0{\cdot}62\,n/N, \qquad r^2 = 0{\cdot}97 \quad . \quad . \quad . \quad (7)$$

(*c*) *The monthly means*

The monthly mean values provide a relation for the five-year period

$$Q/Q_A = 0{\cdot}23 + 0{\cdot}62\,n/N, \qquad r^2 = 0{\cdot}94 \quad . \quad . \quad . \quad (8)$$

5. DISCUSSION

It is clear that no one regression equation can be expected to provide a unique relationship between Q/Q_A and n/N, since the values of a and b have been shown to depend on the manner in which the data are grouped. In general there will have to be selection of the most appropriate equation. Over the five-year period, the general equations are

Mean of a and b populations	$Q/Q_A = 0{\cdot}27 + 0{\cdot}54\,n/N$, $a + b = 0{\cdot}82$ . .	(2)
Daily records	$Q/Q_A = 0{\cdot}26 + 0{\cdot}57\,n/N$, $a + b = 0{\cdot}83$ . .	(3)
Ten-day periods	$Q/Q_A = 0{\cdot}24 + 0{\cdot}59\,n/N$, $a + b = 0{\cdot}83$ . .	(6)
Ten-day periods cross-totalled	$Q/Q_A = 0{\cdot}23 + 0{\cdot}62\,n/N$, $a + b = 0{\cdot}85$ . .	(7)
Monthly periods	$Q/Q_A = 0{\cdot}23 + 0{\cdot}62\,n/N$, $a + b = 0{\cdot}85$ . .	(8)

and there is no statistically significant difference between the regression coefficients.

Since the range of the daily records of n/N is from zero to one, no extrapolation is involved in giving n/N the two limiting values, zero and unity. Consequently the value of a, and the sum $a + b$, will be valid estimates of the threshold value and of the apparent transmission coefficient

(t^m) respectively. With increased smoothing, the extreme values involved in the regressions ar naturally somewhat contracted, so that substitution of limiting values of n/N entails extrapolatior Hence for any grouping other than the daily records the threshold value a and the apparer transmission coefficient t^m will not be as good estimates as those derived from the daily record However, since the estimates of apparent transmission coefficient derived from the variou groupings do not vary very widely, each equation may be assumed to provide a reasonable estimat which can be used for comparison with data derived in a similar manner elsewhere.

Black *et al.* (1954) publish data based on at least 48 individual monthly values for five station which had Campbell-Stokes instruments and which ranged from 30° to 50° in latitude. Super ficially the values of a and b differ from those obtained in Kenya. If, however, it is assumed that th sum of the published mean values of a and b gives an estimate of the mean apparent transmissio coefficient at the different stations, it may be shown that even for these few records the value o t^m is very significantly related to the optical air mass by the equation

$$t^m = 0{\cdot}99 - 0{\cdot}17\,m, \qquad r^2 = 0{\cdot}90 \qquad . \qquad . \qquad . \qquad (9$$

This infers that if values of t^m are reduced to a standard form by correcting them (for latitud and altitude) to optical air-mass unity, values from all over the world should approximate 0·82 This, indeed, is illustrated in the last column of Table 3 which gives the relevant data. The values for mean optical air mass in column 4 are from the *Smithsonian Meteorological Tables* (1951) Clearly the correction to standard measure has reduced the range of transmission coefficien to insignificance. In other words, behind the apparent agreement or disagreement of the actua numerical values of a and b is concealed a certain uniformity. Thus a latitude effect, as suspected by Black *et al.*, is inevitable, though it may not be readily detected by differences in the individua values of a and b, in the equations, for these can vary by chance as already discussed. However, the sums of a and b will readily reflect latitude differences provided that these are not dominated by atmospheric irregularities. The lack of detailed records from other parts of the world at this station has restricted the scope of this investigation at present.

TABLE 3.

Station	Lat.	Alt. (ft)	Mean optical air mass	t^m	t
Rothamsted	51·8°N	< 500	1·62	0·73	0·82
Gembloux	50·6°N	< 500	1·57	0·69	0·79
Versailles	48·8°N	< 500	1·51	0·73	0·81
Mt. Stromlo	35·3°S	2,550	1·11	0·79	0·81
Dry Creek	34·8°S	< 500	1·22	0·80	0·83
Kabete	1·3°S	6,000	0·81	0·85	0·82

Acknowledgment

We are indebted to the Director of the East African Meteorological Department, for lending us the original Campbell-Stokes sunshine cards used at Kabete for the period 1938-1943, and for providing information about the pyranometer in operation at Kabete during the same period.

References

Black, J. N., Bonython, C. W. and Prescott, J. A. 1954 *Quart. J. R. Met. Soc.*, **80**, p. 231.

East African Meteorological Department 1938-43 'Hourly values of radiation on a horizontal surface at Kabete, Kenya' (Mimeograph).

1957 Private communication.

Fisher, R. A. 1948 *Statistical methods for research workers*, London (Oliver & Boyd).

Hamon, R. W., Weiss, L. L. and Wilson, W. T. 1954 *Mon. Weath. Rev., U.S.W.B.*, **82**, p. 141.

Penman, H. L. 1948 *Proc. Roy. Soc.* A, **193**, p. 120.

1955 *Neth. J. Agr. Sc.*, **4**, p. 9.

Riemerschmid, G. 1940 *Onderstepoort J.*, **15**, p. 343.

Smithsonian Institute 1951 *Smithsonian Meteorological Tables*, 6th Rev. Ed.

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Some observations of bent plumes

By F. K. BALL

Division of Meteorological Physics, Commonwealth Scientific and Industrial Research Organization, Melbourne, Australia

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Summary

Some measurements of the height of the smoke downwind from a 40 kw heat source under conditions of light wind are presented. The results are compared with Priestley's and Sutton's theories. A general non-dimensional solution of Sutton's equation is developed which can be applied to bent plumes on any scale.

1. Introduction

Though several theoretical treatments are available of the motion of a bent-over plume of hot gas, e.g. Bosanquet, Cary and Halton (1950), Sutton (1950) and Priestley (1956), in contrast there are few published observational data. The most satisfactory data are contained in an appendix to Bosanquet *et al.* (1950) and consist of the measurement of the heights of the smoke plumes from four different factory chimneys at various distances downwind. In view of the extreme scarcity of observations of this type it was thought worth while to publish a number of observations on a rather smaller scale made at Aspendale, Victoria. These observations were made comparatively close to the source in conditions of light wind. It is expected that under these circumstances the motion of the plume is dominated by buoyancy. The results are compared with Sutton's and Priestley's theories. Bosanquet's theory apparently does not apply to these particular observations for a reason discussed in Section 3.

2. Method of observation

During the period July 1955 to January 1956 observations were made of the height of the centre line of the plume of smoke rising from four oil burners situated close together on the ground. The burners were of the lard-pail type used for orchard heating. The heights were estimated at 30 ft and 60 ft downwind by comparison with a number of vertical ten-foot poles placed on arcs at these distances from the burners. Each value obtained for the plume height is the mean of observations taken about every 15 sec over a period of either two or three minutes. A cup anemometer mounted 6 ft from the ground was used to give the mean wind speed in the same time interval.

The smoke became too diffuse for reliable measurement at distances much in excess of 60 ft. In addition, conditions were usually too irregular if the wind speed dropped below about 3 ft sec^{-1}. When the wind speed exceeded about 12 ft sec^{-1} the smoke was blown along in contact with the ground making the position of the centre line difficult to estimate. In the latter case buoyancy no longer dominates the motion of the plume, and it is more nearly a case of turbulent diffusion from a ground source. The heat output from the burners was estimated from the rate of consumption of oil and found to be about 10 kw/burner. The burning rate tended to increase slightly with wind speed.

3. Comparison with theoretical predictions

According to Priestley (1956) the motion of the plume in the first or buoyancy-dominated phase is given by the following equations :

u = constant

$$w^3 = \frac{3Qg}{2\pi\rho\, C_p\, \theta_e\, c^2}\left(\frac{1}{z} - \frac{z_0^3}{z^3}\right) + \frac{w_0^3\, z_0^3}{z^3} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where u = horizontal speed of plume = wind speed

w = vertical speed of plume

Q = heat output of source

θ_e = temperature of the environment

c = a dimensionless function of u giving the rate of spread of the plume

$R_0 = cz_0$, where R_0 is the initial radius of the plume

w_0 = initial vertical speed

z = the height above a fictitious point source situated a distance z_0 below the real source.

In the present case $w_0 \doteq 0$ (vaporization of oil produces an initial velocity of only about 5 cm sec^{-1}) whence :

$$\frac{w}{u} = \frac{dz}{dx} = \frac{1}{u}\left[\frac{3Qg}{2\pi\rho\, C_p\, \theta_e\, c^2}\left(\frac{1}{z} - \frac{z_0^2}{z^3}\right)\right]^{\frac{1}{3}} \quad . \quad . \quad . \quad . \quad (2)$$

Integration yields :

$$z = \frac{R_0}{c}\left[1 + \left(\frac{2Qg}{\pi\rho\, C_p\, \theta_e}\right)^{\frac{1}{3}} \frac{4c}{3R_0^2}\left(\frac{x}{u}\right)^{\frac{3}{3}}\right]^{\frac{1}{3}} \quad . \quad . \quad . \quad . \quad (3)$$

where x is the distance downwind from the source. In every case considered here the second term in the brackets on the right-hand side is much greater than unity, hence :

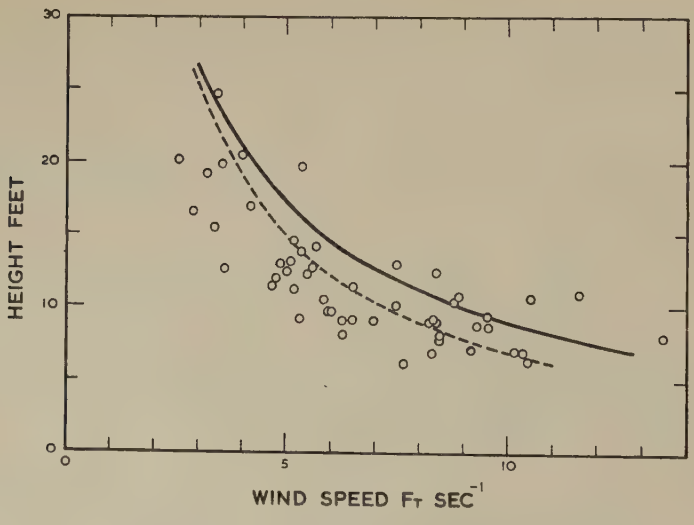

Figure 1. Height of column 30 ft downwind as a function of wind speed.

Priestley - - - - ; Sutton ———

$$h = z - z_0 = \left(\frac{2Qg}{\pi\rho\, C_p\, \theta_e}\right)^{\frac{1}{3}} \left(\frac{4}{3c}\right)^{\frac{1}{3}} \left(\frac{x}{u}\right)^{\frac{2}{3}} - \frac{R_0}{c} \quad . \quad . \quad . \quad . \quad (4)$$

where h is the height above the source.

Priestley suggests that $c \propto u^{\frac{1}{2}}$ and finds from wind-tunnel observations by Halitsky, Randell and Hackman (1950) that $c = 0{\cdot}75$ when $u = 8$ ft sec^{-1}. We have also

$$Q = 40 \text{ kw}, \quad \theta_e = 300°\text{A}, \quad R_0 = 15 \text{ in.}$$

Using this information h has been calculated as a function of u for $x = 30$ ft and $x = 60$ ft and plotted in Figs. 1 and 2 respectively together with the observations. The original observations are given in Table 1. The plotted observations in Figs. 1 and 2 have been modified slightly to take some account of the variation of wind with height near the ground. The method of modification is as follows :

If $h \leqslant 9$ ft, the wind at a height of 6 ft is taken as representative and u is unchanged.

If 9 ft. $< h \leqslant 16$ ft, u is multiplied by 1·10 giving the wind at $h = 12$ ft on the assumption $u \propto h^{\frac{1}{7}}$.

If 16 ft $< h$, u is multiplied by 1·22 giving the wind at $h = 24$ ft on the same assumptions.

According to Sutton the plume obeys the following equation :

$$h = \frac{3}{2}\left(\frac{a}{u}\right)^3 [\cot\psi \operatorname{cosec}\psi - \ln(\cot\psi + \operatorname{cosec}\psi)] \quad . \quad . \quad . \quad (5)$$

where $\tan\psi = \dfrac{dh}{dx}$ and $a^3 = \dfrac{7Qg}{3\pi\, C_p\, \rho\theta_e\, b}$

b is a constant (the generalized diffusion coefficient) given by Sutton as 0·3.

Csanady (1956) has given an approximate solution of this equation and compared the result with the data from Bosanquet *et al.* (1950). His solution is based on the assumption that ψ is small. This assumption is not appropriate to the present case since the measurements were made in the region close to the source where ψ is still appreciable. Accordingly a method has been developed which enables the solution of Eq. (5) to be determined to any desired degree of accuracy. Eq. (5) can be rewritten :

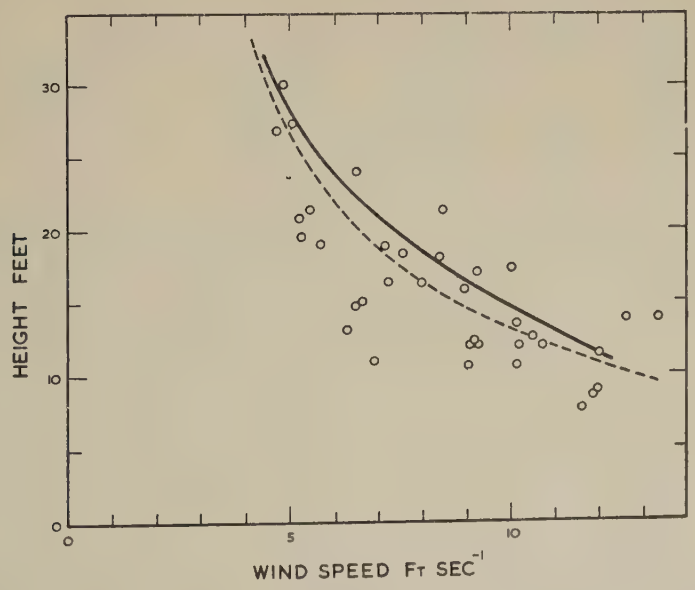

Figure 2. Height of column 60 ft downwind as a function of wind speed.

$$h = A f\left(\frac{dx}{dh}\right)$$

where $A = \frac{3}{2}\left(\frac{a}{u}\right)^3$ and f is a function given by

$$f(s) = s(1 + s^2)^{\frac{1}{2}} - \ln\left[s + (1 + s^2)^{\frac{1}{2}}\right]$$

whence $$\frac{dx}{dh} = f^{-1}(h/A)$$

$$x = a\int_0^{h/A} f^{-1}(s)\, ds$$

Thus $$x/A = F(h/A)$$

where F is a function given by :

$$F(s) = \int_0^s f^{-1}(s)\, ds$$

The function F can be determined graphically or numerically from the known function f. A graph of the function F then gives a general non-dimensional solution of the problem (see Fig. 3). Values calculated in this way are plotted in Figs. 1 and 2.

In order to apply the theory of Bosanquet *et al.* (1950) it is necessary to calculate a quantity which depends on the initial velocity and the rate of emission of gas from the source. These quantities are unknown in the present case, making theory difficult to apply.

Both the preceding theories apply to a region where the wind does not vary with height. In the present case the observations were made near the ground where the wind shear is probably a significant factor. Nevertheless, with only the crude modifications made here both theories give a good approximation to the height of the plume.

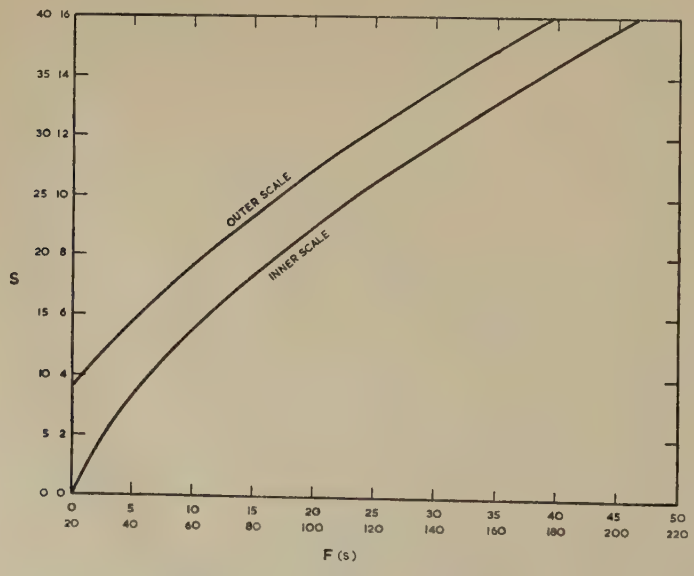

Figure 3. Graph of the function F.

TABLE 1. SUMMARY OF THE OBSERVATIONS

30 ft down wind				30 ft down wind				60 ft down wind			
Date	Time (EST*)	Ht. (ft)	Speed (ft/sec)	Date	Time (EST*)	Ht. (ft)	Speed (ft/sec)	Date	Time (EST*)	Ht. (ft)	Speed (ft/sec)
5.7.55	1015	8·0	13·5	1.12.55	1205	12·9	7·02	4.8.55	0945	13·1	5·87
8.7.55	1015	10·9	10·89	12.12.55	1345	9·1	7·74			14·7	6·10
		8·9	8·37			12·3	7·84	26.10.55	1500	8·5	11·88
12.7.55	1115	9·1	6·07			9·0	6·99			8·9	11·98
20.7.55	1100	20·4	3·48	9.1.56	1510	19·1	2·82			12·6	9·84
		24·6	2·99		1520	11·1	4·89	27.10.55	1140	15·9	8·37
		19·6	4·69	12.1.56	1220	12·6	5·25			10·7	9·51
4.8.55	0935	12·9	4·59		1225	12·3	4·72			13·4	9·51
		9·0	6·27		1237	13·0	4·79			11·4	11·16
15.11.55	0945	9·3	8·92		1242	14·4	4·86	3.11.55	1115	18·3	6·59
		7·0	10·17		1245	10·4	5·45			16·3	6·92
		9·0	8·27		1255	9·7	5·58			10·6	8·46
		6·4	10·47		1340	20·1	2·23			12·0	9·58
		10·6	9·84		1345	15·4	3·15	15.11.55	1440	12·3	8·60
	1435	6·9	8·33		1350	16·5	2·53			17·1	8·04
		7·1	9·17		1355	12·5	4·59			7·7	11·65
23.11.55	1200	8·7	9·58		1400	8·0	6·30			12·0	10·07
		7·0	10·37	13.1.56	1420	11·3	4·40	23.11.55	1205	13·9	11·84
30.11.55	0925	8·0	8·46		1425	11·8	4·49	30.11.55	930	12·0	8·53
		10·6	8·30		1430	16·8	3·65			21·3	7·38
		8·7	9·32	60 ft down wind						19·0	4·95
		11·3	6·14							24·0	5·68
		19·7	3·12	5.7.55	1020	14·0	12·50		1430	15·0	6·23
		13·7	4·99	8.7.55	1025	17·4	8·73			21·4	4·76
		14·1	5·02	12.7.55	1120	18·9	6·23			18·0	7·32
30.11.55	1410	12·1	5·15	20.7.55	1130	16·4	6·30	1.12.55	1230	12·1	8·63
		9·7	5·61			30·0	4·23	12.1.56	1230	20·8	4·54
		9·1	4·95			27·3	4·43		1235	26·8	4·10
1.12.55	1205	10·3	8·24			19·6	4·59		1250	11·0	6·46
		10·0	7·02								
		7·7	8·46								
		6·1	7·64								

* EST (Eastern Standard Time) is approximately 20 min ahead of Local Time.

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REFERENCES

Bosanquet, C. H., Cary, W. F. and Halton, E. M. 1950 *Proc. Inst. Mech. Eng.*, **162**, p. 355.
Csanady, G. T. 1956 *Austr. J. App. Science*, **7**, p. 23.
Halitzky, J., Randell, C. P. and Hackman, M. 1950 *Proc. Inst. Mech. Eng.*, **162**, p. 366.
Priestley, C. H. B. 1956 *Quart. J. R. Met. Soc.*, **82**, p. 165.
Sutton, O. G. 1950 *J. Met.*, **7**, p. 307.

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A preliminary investigation of long waves at Newlyn

By J. DARBYSHIRE
National Institute of Oceanography, Wormley, Surrey

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SUMMARY

Long waves of period greater than ½ min are recorded at Newlyn, Cornwall. There are two predominant types; one with a period of approximately 12 min and another with a period of 30 to 40 min. This investigation deals only with the longer-period type and is concerned with the eight instances during the period Mar. 1956 to Mar. 1957 where these longer waves exceeded 7 in. in amplitude.

The occurrence of these waves appears to be associated with the passage of meteorological fronts and a reasonable correlation is found between the amplitude and the northward component of the frontal velocity whilst the duration of the wave activity appears to decrease as the easterly component of the frontal velocity increases.

Various suggestions regarding the nature of these waves are considered. The evidence that they are edge waves is not very convincing and the most likely explanation appears to be that the waves are due to the oscillation of Mounts Bay as a whole, or of the north-western part of it.

1. INTRODUCTION

Much research has been done recently on sea waves of period ½ min to about 2 hr and their investigation forms part of the oceanographical programme for the International Geophysical Year. Much of the activity within the range of ½ min to 5 min is due to surf beats which are believed to be caused by the acceleration associated with the increasing forward transport of water as ordinary sea waves run on the beach (Tucker 1950). The waves in the region of 10 to 120 min are not so well known and are possibly more interesting from a meteorological point of view. These waves have been studied extensively by Munk and others at the Scripps Institute of Oceanography, California (Munk, Snodgrass and Carrier 1956). From examination of wave records at various points along and off the coast of California, they concluded that the waves behave like edge waves, which can exist when there is a sloping bottom as on a continental shelf. It was shown by Stokes (1846, see Lamb 1932) that such waves were possible and that the wave characteristics were related by the following equations.

$$c = gT \sin \beta/2\pi \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$\lambda = gT^2 \sin \beta/2\pi \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where T is the period, λ the wavelength and β the slope angle. A more exhaustive solution shows that the wave amplitude falls off with the distance from the shore and that it is appreciable only over a distance $\lambda/2\pi$ from the coast. Some of the waves recorded in California could be associated with internal waves on the atmospheric inversion layer. Other waves recorded near Atlantic City could be due to travelling depressions and hurricanes, for if the storm speed were equated to the wave velocity c, and the value of β at each point along the coast used, the period at each point found from the above equations agreed well with the observed values. Munk *et al.* suggested that edge waves could also be caused by the coastal waters being excited by tsunamis, the original incident wave energy being dissipated in the form of edge waves whose length depend on the local topography, resonance effects being obtained in bays and inlets.

A moving pressure disturbance can also affect an area of water of constant depth, h, and if the speed of the disturbance is nearly equal to that of a long wave, $(gh)^{\frac{1}{2}}$, an elevation considerably higher than that due to the static barometric effect can be obtained (Proudman 1954). This effect will also lead to resonance in bays and inlets. For a narrow rectangular bay with two open ends and length l,

$$T = 2l/(gh)^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and for one with one open end

$$T = 4l/(gh)^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

2. Results at Newlyn

A long-wave recorder was set up by the National Institute of Oceanography at Newlyn n Cornwall to study any long waves that might arrive from the Atlantic Ocean. The wave ecorder was developed by M. J. Tucker and P. Booker and was laid at a depth of 4 ft at low prings. It consisted essentially of two pressure transducers backed against each other. The wo have different frequency characteristics, neither responds to ordinary sea-wave periods, nd both have very nearly the same response to oscillations with periods of about 12 hr, so that he effect of tides is very much reduced, but one is much more sensitive than the other to oscilla- ions of 6 to 80 min period, which are thus registered. An example of such a long-wave record aken on 27 to 28 May 1956 is shown in Fig. 1. There is still a residual tidal oscillation but two nain types of oscillation are recognizable, one of about 12 min period and another of about 30 to 40 min period. The 12 min oscillations are very often present and they can be associated with natural oscillations of the northern part of Penzance Bay close to Newlyn, possibly that part of the bay north of the line from Newlyn to St. Michael's Mount. The longer-period waves occur more rarely with appreciable amplitude and must be associated with a larger sea area. The changes in the activity of the 12 min oscillations are interesting and must be studied with the help of continuous records of wind speed and direction, but in the first place it was decided to investigate the 30 min waves.

There were eight instances during the period Mar. 1956 to Mar. 1957 when the 30 min waves reached appreciable amplitudes. These are tabulated in Table 1. The amplitude given is the maximum value obtained during the period.

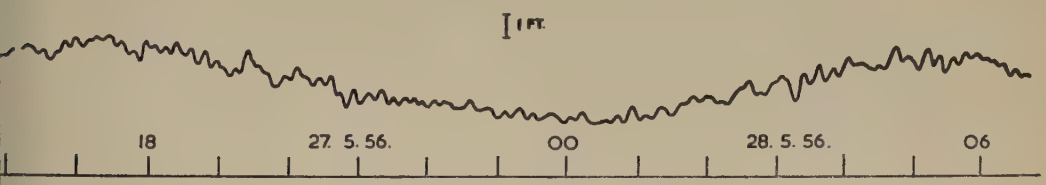

Figure 1. A typical long-wave record.

TABLE 1. 30 MIN PERIOD WAVE OF LARGE AMPLITUDE

Date	Time (GMT)	Amplitude (in.)
27-28 May 1956	1700-0600	12·5
4 July 1956	1300-2200	12·0
15 Dec. 1956	0830-1030	14·4
3-4 Jan. 1957	2200-0400	12·0
30 Jan. 1957	1400-1700	12·0
31 Jan. 1957	0200-1000	9·0
13 Feb. 1957	0000-0600	9·6
6 March 1957	0030-0600	7·2

3. Correlation with meteorological factors

The increase in amplitude was usually preceded by the passage of a front within the previous six hours. It was, nevertheless, difficult to find any correlation between the amplitude and meteorological factors such as the wind speed, wind direction, and temperature changes. The most significant factor seemed to be the speed of the front. The position of a front approaching Newlyn and its position six hours later were plotted and an orthogonal passing through Newlyn was drawn between the two. The northerly and westerly components of the speed could then be estimated from the projections of the orthogonals on the NS and EW lines. This was done for all eight examples. There was a reasonable correlation between the amplitude of the long waves and the northerly component of the front speed as shown in Fig 2 (a). The duration of the long-wave activity is plotted against the easterly component of the frontal velocity in Fig 2 (b). There appears to be a tendency for the duration to decrease with increasing velocity.

The depth of the depression varied widely over the eight examples used but did not appear to have a significant effect.

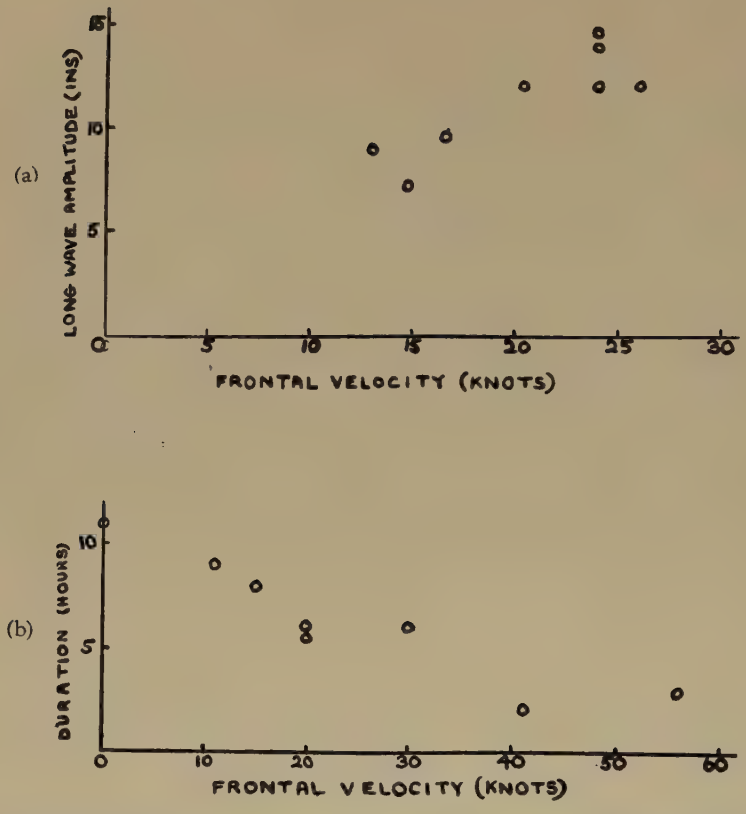

Figure 2. (a) Plot of wave amplitude against the northward component of the frontal velocity. (b) Plot of the wave duration against the easterly component of the frontal velocity.

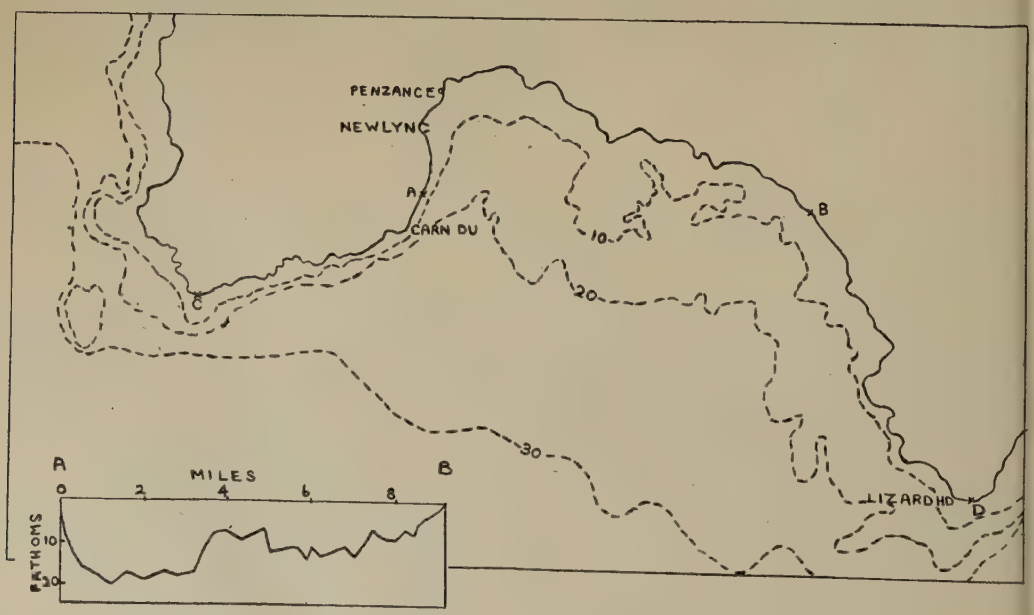

Figure 3. Map and section of Mounts Bay.

4. Nature of the oscillations

Fig. 3 gives a map of Mounts Bay showing the depths and also a vertical section from the points marked A and B on the map. It is seen that the coast at Newlyn is approximately N-S and there is the possibility that the 30 min oscillations are due to edge waves. The distance from Carn Du to Penzance is approximately 4 mi and if the edge wave were a trapped one, resonating in the bay, the wavelength would be about 16 mi and $\lambda/2\pi$ about $2\frac{1}{2}$ mi. The gradient across a strip between the coast and a line $2\frac{1}{2}$ mi out parallel to the coast is assumed to be the same as that shown over the first $2\frac{1}{2}$ mi from A on the section, which is $17\frac{1}{2}$ fathoms in $2\frac{1}{2}$ mi, or 0·007. Using this value in the formula

$$T^2 = 2\pi\lambda/g \sin\beta$$

gives $T = 165$ min, which is larger than the observed value. The velocity c is 87 ft/sec or 52 kt, which is much greater than the highest frontal speeds given in Fig. 2. These results are, however, obviously dependent on the value taken for β and if, for instance, one took a strip between two lines parallel to the coast, at $\frac{1}{2}$ mi and 3 mi from it, the slope would then be 9 fathoms in $2\frac{1}{2}$ mi giving a value of period of the same order as that obtained and a value of velocity comparable to the highest values shown in Fig. 2 (a). The evidence that they are edge waves is not therefore very conclusive.

There remains the possibility that the waves are natural oscillations of the water masses. The obvious possibility is that of Mounts Bay as a whole. It is difficult to calculate the natural period of such an irregularly shaped area of water but an approximation can be made by taking the area north of the line from C to D and considering it as a rectangle of length 10 mi, breadth 5 mi and with a mean depth of 15 fathoms. The period of the N-S oscillation is then given by Eq. (4) and $T = 38$ min. The period of the E-W oscillations is given by Eq. (3) and this also comes to 38 min. The period is of the right order but it is difficult to see why a northward frontal velocity should be more effective than an easterly one as both N-S and E-W oscillations appear to be equally possible. The wave velocity is 53·9 ft/sec or 32·0 kt, which is higher than any of the values given in Fig. 2 (a) but this is not unreasonable as Fig. 2 (a) shows a linear relationship which would not be likely to hold when the velocity values approached the long wave velocity.

Another possibility is that the oscillations are the N-S oscillations of the north-western part of Mounts Bay, west of the southward extension of the 10 fathom line. This is approximately a square of side 4 mi and the mean depth is again 15 fathoms. The natural period given by Eq. (4) is 30 min. The wave velocity is again 32·0 kt. This explanation appears on the whole to be the more probable, as E-W oscillations are less likely to occur, the period is of the right order and the value of the velocity is reasonable.

5. Conclusions

The occurrence of 30 to 40 min period waves at Newlyn, Cornwall can be connected with the passage of meteorological fronts and some correlation is obtained between the amplitude and the northerly component of the frontal velocity. There is also a tendency for the duration to decrease with increasing frontal velocity.

It is not possible with the data available to determine the exact nature of the oscillations. This can only be done by setting up more than one recorder at different points and finding the phase differences. To investigate the existence of edge waves it will be necessary to make measurements off a more regular coast where the effect of local resonances is less.

Acknowledgments

The author wishes to thank the Director and staff of the National Institute of Oceanography for advice and assistance in preparing the paper, and the Harbour Master and staff at Newlyn Harbour for their assistance in taking the wave records.

References

Lamb, H. 1932 *Hydrodynamics*, Cambridge Univ. Press, p. 446.
Munk, W. H., Snodgrass, F. and Carrier, G. 1956 *Science*, **123**, p. 127.
Proudman, J. 1953 *Dynamical Oceanography*, London (Methuen), p. 296.
Tucker, M. J. 1950 *Proc. Roy. Soc.* A, **202**, p. 565.

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On the frequency of snowfall in metropolitan England

By GORDON MANLEY
Bedford College, University of London

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Observation of the frequency of occurrence of snow or sleet depends in large measure o the alertness of the observer. In any event the number of days on which snowflakes are recorde will also depend on whether a watch can be kept throughout the night. For these reasons it i commonly found that a good standard climatological station will record an average annual tota which is about three-quarters of that given by adjacent airfields. In a welcome paper, Ashmor (1952) discussed the reasons for these differences and made a comparison of the annual total over a period of years at a number of stations around the North Welsh border. He was abl to classify them according to quality of observation in five groups, A, A —, B, C and D; an he provided a series of equations linking the annual totals of ' days with snow or sleet observe to fall ' which are likely to be forthcoming from stations in these categories.

It is appropriate to extend these comparisons to the London area and the surroundin countryside, using for reference purposes the 30-year period 1926-1955. A considerable numbe of stations with consistent observations is now available; and since 1930 numerous airfields hav been established at which ' first-class ' standards can be assumed to be maintained. In the Londo area, Kew, Croydon and Greenwich are ' first-class.' At these stations the respective annua averages of ' days with snow ' are 15·6, 18·6 and 17·3. Greenwich observations however only cover 25 years; hence for the whole 30 years, the figure 17·3 has been derived by cross- comparison with the other two stations. The altitudes are 18, 217 and 149 ft. While from time to time slight changes in location, and of altitude, of the instruments have been made at severa stations, of which Croydon is one, for the present purpose these have been disregarded.

There are other stations in and around London at which very high standards are maintained such as Hampstead with an average of 22·8 at 450 ft. Support for the ' first-class ' average of about 16 days close to sea-level is given by Shoeburyness (15·6), equal to the average at Kew. At a greater distance from London, in the Essex lowlands, unusually keen observation is main- tained at Earls Colne (18·0) with an altitude of 168 ft. To the westward, 12 years' observations at Abingdon (210 ft) when reduced, give a 30-yr average of approximately 20 days. Nearby at the Radcliffe Observatory at Oxford (208 ft) an average of 18·7 over 30 years clearly indicates an alertness of observation almost up to ' airfield ' standards. It becomes evident that at the 200 ft level, the expectation lies between 18 and 19 days over a wide area of Southern England. This is supported further afield by South Farnborough (19·5, at 237 ft) and by Cranfield (20·8 at 348 ft, but, like Abingdon, based on 12 years only). Still further to the north-east, but at a lower level, Mildenhall has now 20 years' records, which on reduction give an average for the 30-yr period of 19·5.

In the discussion of such differences two factors, and perhaps three, must be allowed for. The first is altitude. The second is the tendency for additional frequency of precipitation at higher levels. Thirdly, it is possible that the warmth of the built-up area in winter plays some part in diminishing the frequency with which falling snowflakes will be observed in inner London. This effect, in the writer's opinion, must be small.

From the above figures it would appear that where ' first-class ' stations are concerned an appropriate adjustment for altitude is to add 1½ days for each 100 ft above the lowest-level stations, which for convenience have been regarded as 30 ft above sea-level. Now it is broadly true that the climatological stations, at which a constant watch cannot be expected, will as a rule record from two-thirds to three-quarters of the A total. Rothamsted is evidently a particularly good station at the top of this B category, as we might fairly expect. Given this ratio, we should expect the increase with altitude applicable to such stations to be something like 1 day per 100 ft. These factors have been applied in order to provide ' reduced values ' for purposes of comparison. The precise figure to be adopted is open to discussion ; but as far as Southern England is con- cerned, the above reduction factors operate very well.

The accompanying Table 1 is divided into three sections, and shows that the stations can be classified in three groups (A, B, C). The first section comprises the stations in London and its suburbs, together with Shoeburyness. In the second section, stations at a greater distance are given; in the third section, a few stations lying well outside the Thames Basin or at a high altitude are added. The arrangement of the averages in columns draws attention to the classification.

11 A stations, comprising those in the first two sections, adjusted for altitude and including two which are probably A −, give an average of 15·9. 12 B stations, appropriately reduced, give an average of 11·7; and for 8 C stations the average is 8·7.

Ashmore's equations give $N_A = 1{\cdot}1 \quad N_B + 3$

and $N_A = 1{\cdot}25\ N_C + 5$

Applying these, if A = 15·9, B = 11·7 and C = 8·7. The accordance is remarkable. It seems safe to say that most normal climatological stations will fall into the B or C categories and it would appear that, with care, Ashmore's equations can be applied in cases where it becomes necessary to provide data with regard to the frequency with which snow can be expected by ' constant watch.' For example, the relationship Mildenhall-Cambridge is very nearly that required between an A and a B station, which on other grounds one might fairly expect.

In a previous paper (Manley 1940) standardization was effected as far as possible on the best of the climatological stations, i.e. ' good B.' At the time of compilation (1939) relatively few airfield records had become available, but my estimate then made was that ' first-class ' stations might give up to 25 per cent more days with snow, which does not appear to be far wrong.

The relatively short records which we can now use from the upland Chilterns and Cotswolds deserve some mention. Dunstable (516 ft) over 15 yr, has given an annual average which approximates to 22·5 when extended to the 30-yr period. Although this is well above the Rothamsted (420 ft) average of 17·8, it is much the same as Hampstead (450 ft) and by comparison with Cranfield is still rather lower than one might fairly expect, if there were at Dunstable a ' constant-watch ' airfield. But Whipsnade, with 12 yr observations at 720 ft, gives an annual average of 31 days for the 30-yr period. This might be considered as a testimony to first-class standards of observation; but we must not overlook the probable effect of the bold Chiltern escarpment in touching off some additional showers by comparison with the adjacent lowlands. The ' altitude reduction ' is here not the only factor to allow for. Further to the west, the high Cotswolds have a 12-yr ' first-class ' record at Little Rissington (740 ft) which provides us with an approximate value of 27·5 days for the 30-yr average, and a difference from Abingdon which is almost exactly the equivalent of what one would expect from altitude alone. Taking account of the gentle relief, this reinforces one's regard for the efficacy of the reduction of 1½ days per 100 ft. Secondly, we gain support for the view that the additional frequency at Whipsnade must owe much to the effect of the escarpment with regard to polar-air ' instability showers.'

The question also arises of the effect of the ' artificial warmth ' of London. The appearance of the figures leads one to doubt whether this of itself can make more than a very small difference in the frequency with which a really alert observer in a park or other reasonable open space will see snowflakes falling. For the ' heat island ' cannot be deep; and most of the snow is likely to fall on windy days when the ' heat island ' effect will be diminished. No doubt much of what would otherwise fall as snow will become sleet, and it will certainly be much more reluctant to lie on account of the warmth of the ground. But it is to be noted that the climatological stations in Regent's Park and St. James's Park have much the same averages as Writtle, Shinfield, or East Malling, and more than Wisley, Bromley and Southgate, all of which lie higher. These three last-named outer suburban stations give figures comparable with Camden Square or Kensington, when altitude has been allowed for.

No effort has been made to search among the present stations for the equivalent of Ashmore's ' class D.' Suffice it to say that among the discontinued records of past years it might even be possible to declare some as E, but there is no need to discuss them here. Moreover, it is also possible to name at least one former station at which the results of six years' assiduous recording testify to an acuity of observation surpassing that which can normally be expected even at a ' first-class ' station. Clearly, a residuum of uncertainty is always likely to be found, taking account of the existing definition of what is to be recorded, in Gt. Britain, as ' a day with snow.'

Nevertheless, the majority of stations fall into increasingly well-defined categories, and comparisons of this sort can easily be made for other parts of Britain, now that we have a reasonable distribution of airfields maintaining ' constant watch.' Scrutiny of the records continues to show that, here and there, climatological stations have maintained remarkably high standards over lengthy periods; Wakefield is a good example. But there are many reasons why it becomes quite impracticable to expect uniformity of alertness, and in general it is not difficult to estimate the category from the circumstances of the station. All climatological observations have some value, and Ashmore's equations clearly provide an effective method of standardization of the scattered records of an element of considerable climatological importance.

TABLE 1. DAYS WITH SNOW OR SLEET: AVERAGES, 1926-55

Station	Altitude	Length of record (years)	Average annual No. of snow-days 1926-55 (30 yr) (A)	(B)	(C)	Averages: reduced to 30 ft a.s.l. (A)	(B)	(C)
1. London and outer suburbs								
Kew	18	30	15·6			15·6		
Greenwich	149	25	17·3			15·5		
Croydon	217	30	18·6			15·7		
Shoeburyness	11	29	15·6			15·6		
London Airport	82	8	(15·6)			(14·9)		
Northolt	131	6	(17)			(15·5)		
Hampstead	450	29	22·8			16·5		
Biggin Hill	567	11	(24)			(16·0)		
Regent's Park	129	23		11·3			10·3	
St. James's Park	27	25		11·0			11·0	
Camden Square	110	24			9·0			8·2
Kensington	80	30			8·9			8·4
Tottenham	51	16			(8·5)			(8·3)
Bromley	213	24			10·5			8·7
Wilmington	212	10		(14·8)			(13·0)	
Southgate	221	16			(10·8)			(8·9)
Enfield	148	20		13·2			12·0	
Stroud Green	212	9		(15)			(13·2)	
Wisley	150	30			9·9			8·7
Wealdstone	175	13		(14·5)			(13·0)	
2. More distant								
St. Albans	272	16			(11·0)			(8·6)
Ascot-Woolhampton	(340)	18		(15·0)			(11·5)	
Reading (Shinfield)	200	29			11·1			9·4
Reading (Univ.)	148	27		12·5			11·3	
Abingdon	210	12	(20·0)			(17·3)		
Oxford	208	30	18·7			16·0		
Earls Colne	168	30	18·0			16·0		
Writtle	105	15		(11·7)			(11·0)	
East Malling	132	30		11·8			10·8	
West Malling	309	10	(18)			(14)		
Tunbridge W.	351	30		15·1			11·9	
Wye	164	24		12·2			10·9	
3. Upland, and beyond Thames Basin								
Cranfield	348	12	(20·8)			(16·0)		
Dunstable	516	15	(22·5)			(15·2)		
Whipsnade	720	12	(31)			(19·5)		
Rothamsted	420	30		17·8			13·9	
Mildenhall	19	20	19·5			19·5		
Cambridge	41	30		14·9			14·9	
S. Farnborough	237	30	(19·5)			16·5		
Lit. Rissington	740	12	(27·5)			(17·0)		
4. Mean Lowland frequency based on Sections I and II						15·9	11·7	8·7

The letters A, B and C at the head of the columns refer to the 'class' to which each record is allocated as described in the text. The homogeneity at West Malling is a little uncertain and hence this record has not been used in 4. Reductions based on less than 20 years are in brackets. Reduction factor: 1·5 days/100 ft (A) and 1·0 days/100 ft (B and C).

REFERENCES

Ashmore, S. E. 1952 *Quart. J. R. Met. Soc.*, **78**, p. 629.

Manley, G. 1940 *Met. Mag.*, **75**, p. 41.

Correspondence

551.558.1

Air motion over a heated land mass

By Joanne S. Malkus and Melvin E. Stern

We have been greatly interested in a paper of this title by R. C. Smith (1957) and agree that careful studies of prototype problems in variable heating can contribute to understanding the origin and maintenance of atmospheric circulations on all scales. We are particularly impressed by Smith's ingenious resolution of the troublesome upper-boundary condition, which as a result of his efforts should no longer plague workers considering flows over barriers and heat sources.

We were at first puzzled in comparing Smith's results with our own (Stern and Malkus 1953) to note that his motions over a heated island at 0900 hr are so similar to ours for the quasi-steady flow near midday, while his calculated midday flow bore no resemblance to the latter. Resolution of this point has led us to conclude that there is a physical inconsistency in Smith's work which we should like to discuss here.

In choosing the same time dependence for the case where there is a strong basic current U (Smith 1957) and his previous study (Smith 1955) where $U = 0$, Smith states that ' it is reasonable to assume that the heating, by means of insolation and convection, will take a similar form ' in the two situations. It is our contention that this is neither reasonable nor possible for the conditions of the problem discussed.

Magnitude considerations of the first law of thermodynamics lead to the conclusion that the relative phase of the energy supplied to the air by the island and the island temperature depends on the ratio of the time it takes a parcel to cross the island to the diurnal half period. Smith's assumption that the heating function is in phase with the rate of change of island temperature may be justified only if this figure is much greater than unity. In the model under discussion, the island is 200 km wide, $U = 10$ m/sec, and therefore the traverse time (2×10^4 sec) is *less* than the diurnal half period (5×10^4 sec). Maximum heating thus occurs near midday and not in early morning and this would explain the resemblance between Smith's 0900 hr flow pattern and our quasi-steady one for midday.

Actually Smith's 0900 hr temperature relations are not correct for midday because he takes the time rate of temperature change and the heating rate in phase, while our reasoning above shows they are more nearly 90° out of phase. These phase relations were derived more rigorously by Stern (1954). Therefore we suggest that Smith's motion fields are more nearly applicable to later times of day than he has stated, and that his temperature calculations should be re-examined in the light of these phase relations.

Finally, the observations of Charnock, Francis, and Sheppard (1956) and others confirm the points made here in that they found upward, not downward, motions at low levels over a small island in the trades around midday. Our reasoning here suggests that their observations should be qualitatively applicable to 200 km-wide islands.

Woods Hole Oceanographic Institution,
Massachusetts, U.S.A.
12 June 1957.

Reply

By R. C. Smith

In replying, I think I should explain first the domain to which my work applied.

The heating function which was used in the theory represented a heat source which was mainly concentrated in the lowest 500 m of the atmosphere over the heated land mass. It was assumed that the heating distribution was attained by convection from a heated ground layer, the scale of convection having a horizontal magnitude of about 1 km. To avoid this scale of motion the final velocity field was evaluated every 20 km in the horizontal.

Malkus and Stern, however, have concentrated on this lower layer introducing into t equations of motion eddy conductivity terms and arriving at a final vertical velocity field over matter of kilometres where even the position of individual cumulus clouds came out of t mathematics.

To defend my choice of the same heating function in the two cases, except for the variatio in the vertical, I would make the following comments :

(a) The main difference between the two cases is the horizontal advection of the heate air and this is accounted for by the term $U\frac{\partial \tau}{\partial x}$.

(b) In Part II, the heating function is not in phase with the rate of change of potenti temperature in the heated layer, since this is dependent on x, but is in phase with the rate c change of potential temperature when there is no initial wind. Thus the heating is assumed t be independent of the larger-scale motion – a reasonable first approximation.

(c) Some confusion has arisen, I think, from my calculation of rates of change of surfac temperature by introducing $w = 0$ into the equation of added heat. This is a weak point, fo it is in this layer close to the surface that the theory becomes weakest. Here the friction and small scale eddying become important just where the magnitudes of some of the neglected terms in th theory become too large to ignore. Thus the evaluations were only a rough pointer with a larg tolerance in time.

Finally I would like to disagree with the deductions by the authors above from the pape by Charnock, Francis and Sheppard. In their work these latter authors do show downward velocity below a height of 1 km over the island during the day-time.

Meteorological Office,
Air Ministry,
Dunstable, Beds.
21 August 1957.

References

Charnock, H., Francis, J. R. D. and Sheppard, P. A. 1956 *Phil. Trans.* A, **249**, p. 179.
Smith, R. C. 1955 *Quart. J. R. Met. Soc.*, **81**, p. 382.
1957 *Ibid.*, **83**, p. 248.
Stern, M. E. and Malkus, J. S. 1953 *J. Met.*, **10**, p. 105.
Stern, M. E. 1954 *Ibid.*, **11**, p. 495.

551.574.1 : 536.42

Nomenclature of phase transitions

By James E. McDonald

The ambiguity surrounding the meteorologist's use of the term, ' sublimation ' is familiar to all who work or read in the field of cloud physics. After recently completing a paper in which I had to struggle repeatedly with this ambiguity, I resolved to examine the problem systematically and to seek some solution to this annoying nomenclatural problem. The outcome of the latter effort has been summarized by me in some detail elsewhere (' deposition ' – a proposed antonym for ' sublimation,' *J. Met.* (in press)). I wish in this note to reiterate briefly, for the readers of this journal, the recommendation made there : it seems desirable that the meteorologist should follow the physicist in restricting use of ' sublimation ' *solely* to the solid-to-vapour transition, thereby abandoning the long-standing double meaning whereby this term also denoted (in fact, almost more frequently denoted) the inverse transition from vapour to solid. Since physics terminology now embraces only five terms for the six possible phase transitions between the three ordinary states of matter, a sixth term is needed and I urge that the term, ' deposition ' be adopted as specifying the vapour-to-solid transition. In the reference cited, I have given what

I regard as an adequate number of quotations from the literature of meteorology, physics, and chemistry to show that the proposed meaning will not clash with current informal usage and will, in fact, only formalize present loose usage of ' deposition.'

This recommendation is being made by letter to several journals of physics and chemistry in the hope that a uniform terminology for the vapour-solid transitions can be adopted.

In the proposed terminology, the currently used phrase, ' sublimation nucleus ' (which is a direct contradiction in terms to the physicist), would be replaced by the term, ' deposition nucleus,' and one would speak of the ' depositional growth ' of ice crystals as part of the Bergeron-Findeisen process, etc. The circumlocution now required to achieve clarity in discussing the multiphase processes commonly occurring within active rainclouds would be obviated, and the logical goal of having six distinct terms for six distinct phase transitions would be achieved. It is most regrettable that the confusion and ambiguity inherent in what has come to be the meteorologist's use of ' sublimation ' was not avoided years ago when Wegener first coined the now widely used but inappropriate ' sublimation nucleus,' and it is at least odd that it was not corrected long ago by physicists who instead have accepted the confusing practice of using ' condensation ' for transitions from vapour to solid as well as from vapour to liquid.

The proposed change of terminology will belatedly correct this curious difficulty in the nomenclature of phase transitions, and I urge that readers of this journal adopt it.

Institute of Atmospheric Physics,
University of Arizona,
Tucson, Arizona.
18 September 1957.

551.506.3 (41/2)

Early meteorological journals; some further notes

By Gordon Manley

The Society's Librarian has had notice of three more journals, each of which may prove to be of some interest to meteorological historians. The Society gratefully acknowledges the loan of these journals for inspection. Comments follow.

(*a*) *Warwickshire*

Samuel Winter (1703-1787) was a corn merchant of Stratford-on-Avon. In a manuscript notebook, now in the possession of Mrs. L. Merriman, of Reigate, Surrey, he wrote ' a chronicle by way of remembrance of some remarkable things that have happened in my short pilgrimage on earth.' These include a good many descriptive notes on the character of the seasons. For the earlier years 1703-1720 there are a few entries based on such sources as Derham's papers in the *Philosophical Transactions*. From 1739 onward the notes on each year's occurrences become more detailed. Winter appears to have had a considerable correspondence, part of which may have related to his business. One of his most noteworthy remarks concerns the famous frost of 1740; ' the Lake of Windermere is frozen over with so strong an Ice that droves of Cattle and the heaviest Carriages may go safely over.' He mentions such events as the heavy snowfall in the Peak at the beginning of October 1767, the intense heat in July 1750, the very cold May of 1756. Some of his entries refer to events abroad and may well have been culled from one of the well-known monthly magazines. For the most part the floods, thunderstorms, auroras and similar happenings are those of the Midlands, and serve to corroborate or to emphasize the knowledge we already possess based on contemporary instrumental observations. A typescript copy of the entries of meteorological interest has been made for the Society's Library.

(*b*) *Aberdeenshire*

Two further MS records of daily meteorological observations come from Scotland. The first refers to the uplands of Aberdeenshire and is complete for the years 1790 and 1791. It has been handed down in the family of the present owner, Miss M. Usher of Auchnagathle,

by Keig, about 20 mi NW of Aberdeen, to whom the Society is much indebted for the loan of the manuscript. The observations were made by her great-great-grandfather, William Mitchell (1749-1820), who farmed Auchnagathle on the slopes of Bennachie; the prevailing altitude being about 500 ft. They comprise daily notes of the wind direction and the character of the weather kept in ink in a small unbound notebook of 20 pages, 6 in. × 4 in.; the last pages are blank. To these are added numerous notes of farming interest, such as the price of oatmeal and ' bear ' (barley), and occasional marriages and burials.

Entries take the form of a few words for each day, such as ' som frost a fine day continued mod ' for 14 January 1790; and for 18 January ' som soft gray modret all day a showr at night.' The spelling has some unusual features. Frequent use is made of an uncommon word ' roffs ' which may still be used in Aberdeenshire, but I have not been able to trace it*; indeed the first entry (1.i.1790) runs ' frost verry cold some Roffs snow hills WW.' Entries under wind probably refer to the morning and evening direction. For 7 April 1790 we read ' Roffs of snow and haill '; for 14 June 1791 ' very cold roffs ' and for 21 August 1790 ' fine day som Roffs warm all WN.' Hence one has the impression that it meant light passing showers, at any time of year. Another phrase applied to many days is ' strong drought,' which in its context appears to be what we should call a ' good drying day.'

The observer noted the intensity of frost by the thickness of ice on water, $\frac{1}{8}$ or $\frac{1}{4}$ inch as a rule; for 6 December 1791 he enters ' strong frost $2\frac{1}{2}$ inch cold all WSW '; and at the end of that month ' Ice 16 inches in the north water.'

Outstanding are the cold and frequent snow of April 1790 after a mild winter, and the very cold middle of June 1791. ' Snow on the hills ' was noted on the 11, 12, 13 and 22 and ' very cold ' on the 14 and 17. In that year snow fell on 12 June in Northamptonshire, and was said to have fallen in Sussex.

Comments on farming are interesting. ' Oat seeds done ' is entered towards the end of March; and ' bear seed begun ' to 16 April 1791. By 20 October ' harvest a good deal in '; on 10 November, ' some leading '; on 7 December, ' ploughing continued.' In 1791 we read ' harvest begun ' on 20 September, and on 6 October ' dry shearing, fine harvest days.' The late and slow harvest of former days in the upland parishes of Aberdeenshire is well epitomised by these and other entries.

Snow is mentioned fairly often, but it is not easy to disentangle the days of sleet. If we assume that the showery days with snow visible on the hills include sleet at lower levels, we can count 28 days in 1790 and 34 in 1791. (The present-day average by very good observation at Craibstone, 325 ft, is 31). Such an assumption is reasonable, as the hills round Keig are not very high, ranging from 1400 to 1800 ft. There are, of course, more detailed daily instrumental observations for NE Scotland for Gordon Castle, from 1781 to 1827. The interest of this Aberdeenshire diary lies rather in the hint of the difficulties and anxieties of former times. The entries alluding to grey weather from the NE, the strong drying wind from SW, and the cold clear west-wind weather of winter remain characteristic; but today's mechanised farmer has advantages which should ensure that never again will it be recorded that Scotsmen carried the blackened oats from the fields in January, as they were after the dreadful summer of 1782.

(c) *Perthshire*

The third journal was also kept by a farmer, David Wylie, at Busby, which he declares to be 389 ft above sea level, 6 m WNW of Perth. It has been lent to the Society by his great-granddaughter Miss K. Rintoul of Tickenham, Somerset, as a family possession; it covers most of the period between 1823 and 1849. It is a typical earlier nineteenth-century MS in two large note books, comprising carefully ruled and tabulated daily observations of barometer, thermometer and wind, amount of rain, and notes on the weather. The hours of observation were 10 a.m. and 10 p.m. Some letters included with the journal show that it was sent in 1890 to G. J. Symons, who made a number of pencilled annotations. The observations are nearly complete for the year 1823 and again for the year 1826, and thereafter from January 1828 to August 1849. At the end are tabulated the monthly mean temperatures and pressures for the years 1829-49; why the earlier years were not used is not evident. The appearance of the temperatures suggests that they are fairly acceptable shade readings. At the morning observation, extremes of 80° in June 1846 and 3° in February 1838 are noteworthy.

* Miss Usher has enquired of the Editor of the *Scottish National Dictionary*, who locates the word ' raffs ' in Angus, meaning a flying shower of rain or hail.

Most private journals of this type provide indications of character and Wylie for several years indulged himself in quite elegant displays of formal calligraphy. But the most interesting feature is his use of some original and sometimes quite baffling Scottish terms. A good deal of personal enquiry has failed to reveal their exact meaning and it may be necessary to await the completion of the Dictionary of the Older Scottish Tongue. ' Blasty ' for a winter day one can guess at; but whatever the adjective ' scourichie,' ' scourichy ' or ' scourachie ' means I cannot yet discover, save that it seems to appertain more often than not to relatively mild days in the colder months, although it can be found in July and September. Under wind we have ' a breeze,' ' sharp breeze,' a ' gaitlee breeze,' ' blowy,' ' blustering ' and ' boisterous ' as well as ' stormy.' On a June evening in 1830, ' blazning ' ; on a May day in 1847 ' scourie, blaisning.' One evening is ' heavy darke.' In the summer months he quite often enters ' Castles ' which presumably refers to the well-grown cumulus, and is sometimes accompanied by ' sultry.' A variety of other adjectives occur such as ' dribbling,' on a cold July evening, ' hease ' after a morning observation of ' dark showers.' Another evening has ' hot drops '; another has ' halloo dim ' (' halo ' and ' hallo ' are so spelt elsewhere). As with the Aberdeenshire record, we find ' strong drought ' as a descriptive term on a number of summer mornings. Aurora borealis is noted, and one evening in September 1831 ' electrick showers ' appear. ' Harr ' is occasionally mentioned, chiefly in early summer as one would expect. There are a few references to farming; the heavy loss of sheep in February 1838, for example.

The journal may well prove worthy of further study, especially on account of the descriptive adjectives. With regard to the general characteristics of Scottish weather of the period, there is of course plenty of good observational material from Edinburgh, Aberdeen and other centres, and Wylie's rainfall data were long ago investigated by Symons. Neither is it easy to take out frequencies for comparison with stations elsewhere. But someone should elucidate this formidable-sounding term ' scourachie.' On a recent visit to Scotland a Perthshire farmer described the meaning of ' scourie weather ' as ' showers accompanied by wind '; it is possible that ' scourachie ' is an older form.

Bedford College,
University of London.
29 August 1957.

551.5 : 025.4

Note on Universal Decimal Classification

By G. A. BULL
Meteorological Office, Harrow

Some system of subject classification of literature is used in all libraries, except perhaps those dealing solely with novels. There are numerous systems but the one which has found the widest application for detailed classification of scientific literature, in Europe at least, is the Universal Decimal Classification. This classification is controlled by the International Federation of Documentation (F.I.D.), whose office is at the Hague. It is an international body whose President is Dr. Alexander King, late head of the Intelligence Division of D.S.I.R.

The U.D.C. is a hierarchical system going from the general to the particular in decimal steps. Thus 5 is the number under which all the natural sciences are classified. Meteorology as a whole is represented by 551.5. Going further down the scale into detail 551.55 stands for Wind and 551.551.5 for Turbulence in the Free Atmosphere. There are various devices for showing that a paper deals with two subjects independently or with subject A in relation to subjects B, C, etc., that it is in a particular language or deals with some property peculiar to a particular area, say the climatology of London. Any library can use as much or little of the classification as it needs. General libraries whose contents are only books as distinct from articles in periodicals would usually not classify below 'Meteorology.' In a specialized meteorological library every number under 551.5 would be brought into use when needed, but in classifying papers relating meteorology to other subjects, say agriculture or radio-propagation, the full extent of the U.D.C. for the two latter subjects would probably not be necessary.

The present form of the U.D.C. for meteorology is very largely the work of Dr. C. E. P. Brooks in the early 1930's (Brooks 1936). His reorganization was accepted by the International Meteorological Organization in 1935. Since then all papers in this *Journal* have had the appropriate U.D.C. number printed at the head. It is obvious that no system of classification in a developing science like meteorology can remain static. The revision of the system is one of the main functions of the World Meteorological Organization Commission for Bibliography and Publications. The proposals of the Commission, once accepted by the Executive Committee of W.M.O., are submitted to the International Federation for Documentation which examines them from the point of view of the system as a whole and when finally agreed they are published by the W.M.O. in *Technical Regulations*. Meteorology is more fortunate in this respect than some sciences which have no organized body such as the W.M.O. Bibliography Commission to revise the relevant classification. Examination by the F.I.D. is necessary because experts in one subject, left to their own devices, are liable to import into the classification for their own subject one for all the others with which it has any connection.

The last revision of 551.5 took place at the Commission's meeting at the end of 1953, and it was brought into force with effect from 1 January 1957.

The main changes recently introduced are as follows :

551.501.6 : 551.511.1 Thermodynamical diagrams. New number added under 551.501.6 graphical methods of representation – isopleths.

551.508.768 Instruments for measuring ice accretion. New number added under 551.508 meteorological instrumentation.

551.509.313 Dynamics (hydrodynamics and thermodynamics) applied to forecasting. New number added under 551.509 weather forecasting, artificial action on weather. This new number will be used for literature on numerical methods of forecasting.

551.509.314 Statistical methods of forecasting. New number added.

551.509.32 Forecasting of specific elements and phenomena. New number added. This number is sub-divided to cover individual elements (e.g., 551.509.325 covers forecasting of visibility).

551.510.522 Layer in contact with the Earth's surface. New number added under 551.510.52 troposphere. This new number will be used for papers on the general meteorology of the atmosphere at breathing level and heights accessible by fixed instruments.

551.510.528 Tropopause. New number added. Previously there was no number for the tropopause as such and the device of using 551.510.52/551.510.53, i.e., troposphere/stratosphere has been used in the Meteorological Office Library.

551.510.53 Atmosphere above the tropopause (highest tropopause if there is a multiple arrangement). The title of this number was changed from stratosphere in general in recognition of the increase in complexity of the known structure of the higher atmosphere.

551.511 Mechanics and thermodynamics of the atmosphere in general. This number is used for papers dealing with the mechanical and thermodynamical processes in air, regarded

om the theoretical or physical point of view, without reference to the place of the air in any ecific circulation, such as a depression. There is another number, 551.515, with suitable ib-divisions for classifying papers on the dynamics of depressions and anticyclones. It has for ng been recognized that 551.511 needed sub-division to permit easier finding of papers on ecific parts of theoretical dynamical meteorology and the first session of the Commission for ibliography and Publications produced one as follows :

551.511.1 Statics and quasi-statics.
.12 Hydrostatics. Standard atmospheres.
.13 Static and quasi-static thermodynamic states and processes. Thermal equilibrium.
.2 Kinematics.
.3 Dynamics.
.32 Hydrodynamics. This number will be used for theoretical papers on such ubjects as the general equations of motion, variation of wind with height, geostrophic relation, nd the conservation theorems of hydrodynamics.
.33 Thermodynamics. This number will be used only for papers on subjects in hich variation in time plays an essential part.
.6 Turbulence and diffusion. This number will be used only for papers which re not wholly or mainly theoretical. Papers which are wholly or partly observational will be iven either 551.551 turbulence, gustiness, turbulent diffusion or both 551.551 and 551.511.6.

551.521.326 Radiation from the night sky. New number under 551.521.32 terrestrial and tmospheric radiation in place of 551.521.4 which has been deleted.

551.547.5 Distribution in space (of upper-air pressure), isohypses, isobars. New number dded under 551.547 upper-air pressure.

551.551 Turbulence, gustiness, micro-variations of wind, turbulent diffusion in the tmosphere has been renamed from wind structure, micro-variations, gustiness, turbulence, nd is sub-divided as follows :

551.551.2 Turbulence in the lower layers defined as the region accessible by instruments mounted on the ground, on masts, towers, and fixed balloon cables.
.21 Turbulence in the layers up to normal anemometer height.
.25 Turbulence in the lower layers above normal anemometer height.
.5 Turbulence in the free atmosphere.
.8 Turbulent diffusion of momentum, heat, water vapour and aerosols.

551.556.6 Wind pressure on buildings. New number added under 551.556 effects of wind, n place of 551.556 : 69 which has been deleted.

551.556.8 Effects of wind on water surfaces. New number added.

551.557.5 Specific large-scale winds (e.g., jet stream, anti-trades). New number added nder 551.557 upper-air wind.

551.589.6 Quasi-periodic phenomena (e.g., Ice-Saints' Days). New number added under 551.589 synoptic climatology.

551.593.653 Noctilucent clouds. Name changed from luminous clouds.

This note is published with the agreement of the British Standards Institution, copyright nolders of the Universal Decimal Classification in the United Kingdom. A list of the relevant B.S.I. publications is included below.

References

British Standards Institution, London 1943 *Universal Decimal Classification*, English Edition, British Standard, 1000 **2**, Pt. 3, Geology (including Meteorology).

1957 *Ibid.*, Abridged English Edition (revised enlarged with detailed alphabetical index). British Standard 1000A.

Brooks, C. E. P. 1936 ' The classification of meteorological literature', *Quart. J. R. Met. Soc.*, **62**, p. 134. (This article describes the new U.D.C. 551.5 for meteorology devised in 1935 by Dr. Brooks which has since formed the basis of classification.

International Federation for Documentation (F.I.D.), The Hague *Extensions and corrections to the U.D.C.* Issued half-yearly cumulative.

Meteorological Office, London 1950 Universal Decimal Classification, *Met. Mag.*, **79**, p. 59. (Note describing the changes in 551.5 introduced after international meetings at Berlin in 1939 and Toronto in 1947).

World Meteorological Organization, Geneva 1956 *Technical Regulations*, **1** – General, Annex V., W.M.O., No. 49. BD.2.

Proceedings at meetings of the Society

Ordinary Meeting, 16 October 1957

The following candidates were declared to have been elected by the Council during th summer recess :

Fellows

Bramble, Henry James, 54 Chatsworth Avenue, Cosham, Portsmouth.
Cheema, Bashir-Ud-Din, B.Sc., Department of Plant Protection, Pakistan Secretariat Block 5 Karachi, West Pakistan.
Clarke, Michael Martin, 17 Harvard Road, Chiswick, London W.4.
Coe, David John, Instr. Lt., B.Sc., R.N., H.M.S. *Ganges*, Shotley Gate, Ipswich, Suffolk.
Dent, Lawrence, B.Sc., c/o Officers' Mess, R.A.F. Bahrain, Persian Gulf, B.F.P.O. 63.
Fry, Gerald Edward, Cromer House, 42 York Street, Rugby, Warwicks (*transfer from Studen Membership*).
Hannell, Francis George, Ph.D., Department of Geography, The University, Bristol 8 Somerset.
Kumar, Satyaindar, M.Sc., Sudan Meteorological Service, Khartoum, Sudan.
Morgan, Robert Henderson, Instr. Cdr., M.A., B.Sc., R.N., c/o C.W. Branch, Admiralty London, S.W.1.
Oxley, Wilfred John, 119 Calder Road, Maidstone, Kent (*transfer from Student Membership*)
Stanbury, Alan Favell, A.C.I.S., 62 Hinstock Road, Handsworth, Birmingham 20, Warwicks
Talbot, Mrs. Noreen, B.Sc., 62 Ferndene Road, London, S.E.24 (*transfer from Studen Membership*).
Unwin, Thomas Eric, 56 Pickhurst Park, Bromley, Kent (*reinstatement; elected 1945, resigned 1952*).
Welburn, Thomas Hugh Norton, B.A., School House, Dovercourt, Harwich, Essex (*transfer from Student Membership*).
Wiesner, Clarence Jack, School of Civil Engineering, New South Wales University of Technology, Broadway, N.S.W., Australia.
Witham, Julia Cicely, Geraldton, Ash Lane, Emerson Park, Hornchurch, Essex.

Foreign Member

Santomauro, Prof. Dr. Luigi, Osservatorio Astronomico Di Brera, Via Brera, 28, Milan, Italy.

Student Members

Adams, Terence David, 10 Foundry Road, Skewen, Neath, Glamorgan.
Brock, Andrew, B.A., 11 Shenfield Crescent, Brentwood, Essex.
Crowder, Robert Antony, ' Whyte Ridge,' Queen's Road, Devizes, Wilts.
Fox, Jeremy James, ' Winterset,' Pilgrims Way, Guildford, Surrey.
Griffiths, Roy Peter, 66 Dene Road, Didsbury, Manchester 20, Lancs.
Hardy, 4182369 S.A.C. Norman Edwin, Comet Servicing Sqdn., R.A.F. Lyneham, Nr. Chippenham, Wilts.
Martin, Brian, 44 Pitville Road, Mossley Hill, Liverpool, 18, Lancs.
Mayne, Gabrielle Maxine, ' Quiet Ways,' 17 Albany Terrace, Britannia Square, Worcester.
Porter, Malcolm, The School House, Melmerby, Penrith, Cumberland.
Reason, John Martyn, 40 Northcourt Avenue, Reading, Berks.
Reid, Pamela Mary, 65 Blackbrook Lane, Bickley, Bromley, Kent.
Smythe, Charles Richard, 86 Ridgeway, Weston Favell, Northampton.
Sutcliffe, Peter William, 42 Newlands Lane, Workington, Cumberland.
Watson, Neil Rae, 90 Rushford Street, Manchester 12, Lancs.

Canadian Branch

Fellows

Lemire, Francois Jacques, B.Sc., M.A., 315 Bloor Street West, Toronto 5, Ontario, Canada.
Orvig, Svenn, M.Sc., Ph.D., Department of Geography, McGill University, Montreal P.Q., Canada.

The following candidates were then declared to have been elected that day by the Council :

FELLOWS

ARRY, Roger Graham, B.A., 37 Harley Road, Ecclesall, Sheffield 11, Yorks. (*transfer from Student Membership*).
ILLS, David John, Instr. Lt., B.Sc., R.N., ' Montrose,' Redhills, Exeter, Devon.
ATH, Paresh Chandra, B.Sc., c/o Air India International Operations Department, Santacruz Airport, Bombay 29, India.

STUDENT MEMBERS

ANLEY, Gordon Alfred, 27 Meredith Road, Portsmouth, Hants.
AGG, C. J., Wayside Cottage, 412 Woodham Lane, West Byfleet, Weybridge, Surrey.

FOREIGN MEMBER

UEDES, Carol J., B.Sc., Geophysical Institute, University of Alaska, Box 47, College, Alaska.

Dr. C. H. B. Priestley, Vice-President (Head of the Division of Meteorological Physics, Commonwealth Scientific and Industrial Research Organization, Victoria, Australia) spoke on he work of his Division and answered questions.

Special General Meeting, 20 November 1957

Professor P. A. Sheppard, President, in the Chair and about 60 Fellows present. A Special General Meeting of Fellows was held to consider the following motion submitted by Council :

' To amend By-Law 36 by deleting the stated annual subscriptions and substituting :

Fellows - - - -	£4 4s. 0d.
Foreign Members - -	£4 4s. 0d.
Associates - - -	£3 0s. 0d.
Students - - - -	£1 10s. 0d.

The notice, dated 28 October 1957, circulated to Fellows to call the Meeting, contained he following explanatory note :

' The Officers, in conjunction with the Finance and General Purposes Committee, have ecently given special consideration to the financial position of the Society.

Over the past few years it has become increasingly difficult for the Society to pay its way vhile at the same time maintaining a high level of scientific activity. During the current year wo events have combined to cause a decisive deterioration in the Society's finances : The Commissioners of Inland Revenue have refused to grant a rebate on covenanted subscriptions, nd the annual grant made by the Royal Society to the general fund has been withheld. It would ppear unwise to rely, in the future, on either of these sources of income.

It was estimated that these two factors, together with the general economic trend, will involve he Society in an adverse balance of approximately £1,300 per annum.

The Officers reported on the position to Council at its meeting on 16 October 1957 and it vas then decided to recommend to the Fellowship that the subscription of members should be ncreased. Council greatly regrets that this step should be necessary, but feels that no other course s practicable. Members will recall that the present subscriptions were laid down in 1947 in inancial circumstances very different from those that exist today.'

The motion was proposed by the President, who said that only a few letters commenting on he proposed increases in subscriptions had been received, some of a favourable nature. He xplained that special cases of hardship resulting from the proposed increase in subscriptions vould receive the sympathetic consideration of Council, which was empowered to deal with them n accordance with By-Law 28. In saying this he had especially in mind those Fellows of long tanding who may have retired on pension some years ago.

After discussion the motion was put to the vote and carried with only two dissentients.

Ordinary Meeting, 20 November 1957

The following candidates were declared to have been elected that day by the Counci

Fellows

Brinkman, Stanley Ernest, P.O. Box 7303, Nairobi, Kenya.
Clarke, Reginald Harry, B.A., B.Sc., 123 Mountain View, Parade Rosanna N22, Victori Australia (*transfer from Associate Membership*).
Cooper, Captain Hubert Ronald, ' Claremont,' 118 Boness Road, Grangemouth, Stirlingshi Scotland.
Haberfield, John Denis, 30 Central Avenue, Pinner, Middx.
Huckle, Vera May, B.Sc., 13 Dale Road, Dunstable, Beds.
Johnson, Michael Andrew, M.A., B.Sc., Ph.D., 12 Highgrove Avenue, Chilwell, Notts. (*transf from Student Membership*).
Partis, Thomas William, 7 St. Neots Road, Harold Hill, Romford, Essex.
Russell, Allan Sharp, D.F.C., Officers' Mess, R.A.F. Bahrain, Persian Gulf, B.F.P.O. 63.
Shipp, Arthur James, Captain, c/o W.A.A.C., Ikeja Airport, Lagos, Nigeria.
Sinha, Keshab Lal, M.Sc., Meteorological Officer, Gauhati Aerodrome, Assam, India.
Telford, Godfrey Randolph, 192 Petersham Road, Marlborough, Salisbury, S. Rhodesia.
Walker, Harry Owen, B.Sc., Director, Ghana Meteorological Department, Box 744, Accr Ghana.
Wilson, John William, Lamer Lodge, Wheathampstead, Herts.

Foreign Members

Feige, Joachim Helmut, Kiriat Moshe 53, Petach-Tiqua, Israel.
Huneidi, Abdul Latif, Meteorological Officer, Ministry of Communications, Civil Aviatio Amman, Jordan.
Slade, David Hammer, B.S., M.A., 833 Central Avenue, Asbury Park, New Jersey, U.S.A.

Student Member

Masser, Francis Ian, Wayside, Broughton Road, Malton, Yorks.

Canadian Branch

Fellow

Eddy, Amos, B.A.Sc., M.A., Central Analysis Office, Meteorological Branch, Trans-Atlanti Administration Building, Montreal Airport, Quebec, Canada.

Associates

Brun, Paul Robert, B.A.Sc., 315 Bloor Street West, Toronto 5, Ontario, Canada.
Henderson, John, B.A., 315 Bloor Street West, Toronto 5, Ontario, Canada.
Parkinson, David Harvey, B.A., 315 Bloor Street West, Toronto 5, Ontario, Canada.
Shakir, Ali Memet, B.A., 315 Bloor Street West, Toronto 5, Ontario, Canada.
Turner, Vernon Raymond, 315 Bloor Street West, Toronto 5, Ontario, Canada.
Warr, J. R. F., M.Sc., B.Ed., 315 Bloor Street West, Toronto 5, Ontario, Canada.

The following papers were read and discussed :

1. ' Jet streams and long waves in a steady rotating-dishpan experiment : structure of the circulation ' by H. Riehl and D. Fultz (*Q.J.*, **83**, p. 215).

2. ' Evidence of a mean meridional circulation in the atmosphere from surface wind observations ' by G. H. Tucker (*Q.J.*, **83**, p. 290).

Ordinary Meeting, 18 December 1957

The following candidates were declared to have been elected that day by the Council :

Fellows

RITISH SOCIETY FOR RESEARCH IN AGRICULTURAL ENGINEERING, Wrest Park, Silsoe, Beds., (*Nominee* : John Victor Lake, Ph.D.).
ULLEN, Joseph Francis Michael, Lt., 10 Linton Road, Shoeburyness, Essex.
ARD, Wilfred Ingram, M.D., F.R.C.P.E., F.R.C.P., 6 Netherby Road, Edinburgh 5, Scotland.
ANSING, Derek Albert, 2 Malford Court, South Woodford, London, E.18.
ONES, Alfred Stanley, ' Oaklands,' Plough Lane, Christleton, Chester, Cheshire.
EADEN, George Terence, B.A., ' Southleigh,' Frome Road, Bradford-on-Avon, Wilts.
AJAGOPALAN, Perinchery Kulappura, c/o s.s. *India Endeavour*, India Steamship Co., Ltd., 21 Old Courthouse Street, Calcutta 1, India.
VHITAKER, Charles Trimby, 14A Charlotte Street, South Shields, Co. Durham.

Student Members

AKIMI, Abdul H., Department of Meteorology, Imperial College, Huxley Buildings, Exhibition Road, London, S.W.7.
ACKLIN, William Charles, 34 Castlebar Road, Ealing, London, W.5.
TUBBS, Martin William, 61 Allangate Road, Liverpool 19, Lancs.

Canadian Branch

Fellows

AN LEEUWEN, Hans Cornelius Thomas, B.Sc., 315 Bloor Street West, Toronto 5, Ontario, Canada.
VESTWICK, Henry, B.A., 315 Bloor Street West, Toronto 5, Ontario, Canada.

Associates

URSLEM, Michael Rashleigh Grant, B.Sc., 315 Bloor Street West, Toronto 5, Ontario, Canada.
OHNSTONE, Kenneth Russell, B.Sc., 315 Bloor Street West, Toronto 5, Ontario, Canada.
ICCOLM, Kenneth Edward, B.Sc., 315 Bloor Street West, Toronto 5, Ontario, Canada.
TRANG, David William, B.Sc., 315 Bloor Street West, Toronto 5, Ontario, Canada.

The following papers were read and discussed :

1. ' Dew ' by J. L. Monteith (*Q.J.*, **83**, p. 322).
2. ' Water losses from various land surfaces ' by N. E. Rider (*Q.J.*, **83**, p. 181).

Popular Lectures

14 November 1957. The President, Professor P. A. Sheppard, being in the Chair, Mr. H. H. Lamb gave the first of the Society's series of Popular Lectures for 1957-1958, his subject eing ' Weather in the polar regions.' The Lecture was open to sixth-form pupils in the London rea and was held in the Conference Hall of the London County Council by kind invitation of he Chief Education Officer of the L.C.C.

26 November 1957. Mr. J. B. Rigg being in the Chair, Mr. Lamb repeated his Popular ecture, which was again held in the Conference Hall of the London County Council. By rrangement with the Education Officers of Buckinghamshire, Essex, Kent, Hertfordshire and urrey, pupils from these areas were present, together with a number of pupils from independent chools.

The total attendance at the two lectures at the Conference Hall was about 850.

Reviews

Cloud study : a pictorial guide. By F. H. Ludlam and R. S. Scorer. London (John Murray), 1957. Pp. 80 7 Figs.; 73 Plates. 12s. 6d. (11s. to Members).

It is a very pleasant occupation to sit and look at clouds and occasionally to photograph them. Thi pleasure can be increased considerably if one is able to relate their appearance and development to th physical factors producing them and perhaps use this understanding to make a personal short-term forecas of the local weather. The purpose of this book, which was compiled under the auspices of the Roya Meteorological Society and contains a foreword by the President, is to describe the common forms of cloud and show how their relation to weather conditions can be recognised. The first part consists of a short accoun of the atmospheric processes which are associated with clouds, and describes their forms and composition Although designed for the reader without special knowledge, this can be read with profit by all interested in meteorology. The book includes over 70 cloud photographs, several of them in colour and all excellently reproduced, selected from many sources including the Clarke and Cave collections, Royal Air Force photographs, and those taken specially by the authors. Their quality and interest may be compared not unfavourably with the classics of the *International Cloud Atlases* of the World Meteorological Organization.

The authors, both well known for their work in cloud dynamics, have by reference to their introductory section and by explanatory notes alongside the photographs given a detailed description of the accompanying physical conditions. Understandably these are their own interpretations and the selection and emphasis has been obviously influenced by their own researches. This does not detract at all from the book, however, as they have undoubtedly achieved their main purpose of stimulating the reader to observe clouds and think about their formation.

It is to be hoped that with its remarkably low price and popular appeal a very wide circulation will be achieved. Certainly it is a very useful reference book for all meteorologists, professional and amateur. It also deserves consideration by Educational Authorities for use in schools and amongst others by Public Libraries for the casual reader who is very likely to be attracted by its purely artistic content.

R.J.M.

Artificial stimulation of rain. Proceedings of the First Conference on the physics of cloud and precipitation particles. Eds. H. Weickmann and W. Smith. London (Pergamon Press), 1957. Pp. xvi, 427; many Figures and Plates. Five guineas.

The main title of this book is misleading. It contains the proceedings of a conference on the physics of cloud and precipitation particles, which was held at Woods Hole, Massachusetts, in September 1955, and which was attended by over 70 scientists working in this field. Their individual studies can all be regarded as more or less related to the eventual achievement of rain stimulation and other kinds of weather control, but everyone realizes that this is a very distant goal, and most of the papers in this volume make no direct reference to it.

The papers are grouped under four main headings : Aerosols; Cloud droplets and rain from droplet clouds; Freezing and its part in rainfall generation; and Laboratory and field studies of crystal nucleation and growth. There is thus an imposing array of subjects, all properly part of cloud physics. There is a disappointing lack of cloud dynamics; unless this absence is squarely faced and remedied one feels there will be slow progress with atmospheric problems.

Nevertheless much interesting and fundamental work is reported and commented upon. Although the editors have arranged the papers well and ensured that the oral discussions have more coherence than usual, it is of course beyond him to weld the miscellany into any kind of a structural whole which could be represented by a simple title. One may enquire whether it is justifiable to publish in this style the proceedings of conferences which now tend to succeed each other yearly, with some repetition of content. On the one hand the book is beautifully if not extravagantly produced, in the manner which all scientific work should surely deserve. On the other hand the consequent price is so high that many will wish that the more substantial papers could have appeared in the usual journals, rather than be faced with a harrowing decision on whether or not they can afford the book.

F.H.L.

Weather analysis and forecasting. Vol. II. *Weather and weather systems.* By Sverre Petterssen. New York (McGraw-Hill), 2nd Edition, 1956. Pp. xii, 266; 170 Figs.; 33 Tables; 7 Plates. $6.00 (45s.),

The first volume of this new work, a complete rewriting of that which appeared with the same title 17 years ago, was on *Motion and motion systems* and was noticed earlier (*Q.J.*, **82**, p. 543). It was a much needed and successful volume in spite of the fact that much of the subject matter was new and is still being vigorously developed. This second volume, *Weather and Weather Systems* aims to round off what is stated on the jacket to be ' a very complete text on forecasting.' This is to claim too much and I shall list the chapter titles as the best clue to the contents : Production and Transformation of Air Masses, containing a discussion of vertical eddy fluxes but not of radiation flux : Temperature and Humidity Relationships – the classical elementary ' thermodynamics '; Clouds and Precipitation, mainly a selection of photographs with descriptive text and a little cloud physics; Quantitative Precipitation Forecasting, a relatively difficult dynamical chapter by J. C. Thompson; Mist, Fog and Stratus, an elementary descriptive chapter; Convective Clouds

and Weather, a valuable treatment, much of it from recent research; Weather Systems, mainly about classical synoptic analysis with some examples; Local Forecast Studies, a special little chapter on a statistical technique which has been used in practice in America; and finally, Application of Synoptic Climatology to Weather Prediction, a short, specialized essay by T. F. Malone on a neglected subject.

It will be agreed, I think, that here we have a number of relevant but rather disconnected essays; not a systematic text-book on forecasting. There is also some evidence of hasty publishing, a number of misprints, and some index omissions (e.g. mesoscale analysis, radar). But whatever criticisms may be made the book is likely to be valued by beginners in forecasting, especially those with a limited background of physics and mathematics; it should certainly find a place in the forecaster's library alongside the more valuable Volume I.

R.C.S.

The atmospheric diffusion of gases discharged from the chimney of the Harwell Pile (B.E.P.O.). A.E.R.E. HP/R 1452. By N. G. Stewart, H. J. Gale and R. N. Crooks. London (H.M.S.O.), 1957. Pp. 40; 23 Figs.; 10 Tables. 9s.

As a study of the distribution of airborne material downwind of an elevated source the work described in this report is especially notable in two respects. In the first place it was concerned with a radioactive effluent and the distribution was in fact measured in terms of the beta and gamma activity of argon 41. In the second place a good deal of the survey effort was devoted to measurements in a *vertical* plane, using instruments mounted on the cable of a captive balloon.

The vertical surveys were carried out to a height of 300 m at distances up to 1,000 m downwind of the 68 m chimney. Ground-level surveys, providing directly the quantities of ultimate interest, covered distances up to 10,000 m. The two main sections of the report give a detailed account of the conditions, procedure and results of these surveys, and are preceded by brief discussions of the method of measurement and of the principal micrometeorological factors involved.

The interests of most meteorologists who read this report will no doubt be particularly engaged by the vertical measurements and the indications thus provided of the rise and vertical spread of the effluent plume. Altogether 53 successful surveys were analysed and the data on vertical spread are presented both directly in the form of values of σ_z (the standard deviation of the vertical distribution) and indirectly in the form of equivalent values of the virtual diffusion coefficient, C_z, in Sutton's diffusion formulae. Systematic variations of σ_z with distance, wind speed and the vertical gradient of temperature are displayed. The plume-height observations show variations with distance, wind speed and temperature gradient in the expected general sense.

The analysis contains recognition of two features which are not always properly appreciated in studies of the present type. Because of the wide spectrum of eddy sizes in the atmosphere the distribution from a continuous source is noticeably a function of the time over which the distribution is averaged. For most of the present measurements the sampling times were in the region of 20 min but a number of special surveys demonstrated a slow increase of plume spread with increase in sampling time. The other feature is that in common with most studies of effluent dispersion the diffusion near the ground level is affected by local turbulence generated by buildings. In the present report this latter effect is considered to be reflected in the decrease of the apparent diffusion parameter at the greater distances from the chimney.

The report is well written with the results clearly set out in tabular and graphical form, and the authors are to be congratulated on a study which combines the practical and the fundamental outlooks to a nice degree. That these unique observations of diffusion did not have any measurements of turbulence to go with them requires no criticism of the work but merely reflects the traditional reliance on more convenient indicators such as temperature gradient and wind gradient. It is to be hoped that future studies of this type will include those measurements of turbulent fluctuations which have since become increasingly practicable and which may well subscribe more effectively to a better understanding and wider application of the results.

F.P.

The Planet Earth. Ed. D. R. Bates. London (Pergamon Press), 1957. Pp. 312; 47 Figs.; 31 Tables. 35s.

I thoroughly enjoyed reading this book and shall find it worth picking up many times again as giving what I would take to be an up-to-date impression of many things that geophysicists are thinking about in this year 1958. It is aimed, and very successfully on the whole, at the level of the general reader with scientific interests, but is far removed from popular scientific journalism and will be profitable reading to many geophysicists in fields outside their specialization. With the scope as wide as it is and an average of only 17 pages allotted to each topic, it is hardly justifiable to comment in detail. A list of chapters and authors will do more to indicate the value of the book than any summary a reviewer could make.

S. Chapman, F.R.S., The International Geophysical Year; G. P. Kuiper, Origin, Age, and Possible Ultimate Fate of the Earth; K. E. Bullen, F.R.S., The Deep Interior; J. T. Wilson, The Crust; G. E. R. Deacon, F.R.S., The Oceans; E. H. Vestine, Geomagnetic Field; D. R. Bates, F.R.S., Composition and Structure of the Atmosphere; E. T. Eady, Climate and The General Circulation of the Atmosphere and Oceans; E. J. Öpik, Ice Ages; B. J. Mason, Meteorology; J. A. Ratcliffe, F.R.S., The Ionosphere; D. R. Bates, F.R.S., The Airglow; V. C. A. Ferraro, Aurorae and Magnetic Storms; A. C. B. Lovell, F.R.S., Meteors; J. G. Wilson, Cosmic Radiation; J. B. S. Haldane, F.R.S., Genesis of Life.

There is a galaxy of talent which could not fail to be interesting and it does not. What, taking the book as a whole, I admire more than anything, is the straightforward style presenting a great deal of information, more or less factual, more or less speculative, but ' authoritative,' suitably interwoven with discussions of techniques and problems. I have never been one to join the popular chorus in decrying the powers of expression, in the written word, of my scientific colleagues and this compendium reinforces my good opinion.

Naturally I might be expected to turn my attention primarily to the meteorological chapters although these are not the ones which meteorologists will most require. Eady on the General Circulation is good and the linking of atmosphere and oceans in one chapter is an excellent feature; his chapter on Climate seems, by contrast, to lose an opportunity to present the subject as a living science owing to an almost philosophical preoccupation with the limitations of empiricism. Perhaps the oddest thing about Mason's chapter on Meteorology is its title. Its 29 pages provide 19 on general weather forecasting and 10 on the physics of clouds and rain-making. One has no quarrel about the importance of these things but in the scientific company of this compendium it is sad that space could not be found to refer to many more of the physical problems of the atmosphere which are exercising the skills and imaginations of meteorologists at the present time.

R.C.S.

The United Kingdom contribution to the International Geophysical Year 1957-58. The Royal Society, 1957. Pp. 72; 7 Figs.; 21 Tables; 1 Plate. 10s.

In both its impressive appearance and its painstaking detail, this book is a cousin of the *Royal Society Yearbook*. For those who wish to know details of the British committees, observing stations and equipment concerned in the I.G.Y. this book will be the authoritative source. It is however not likely to excite much interest of a more general nature, for there are no details of the kind to interest and satisfy the scientist of a different discipline, nor, despite the careful definitions of technical terms, is it styled to suit the layman.

It may perhaps be a subjective matter, but the reviewer felt that some of the sections were written from restricted viewpoints, sometimes giving misleading impressions of the state of the subject. For example in Chapter 1 we find that ' The main purpose of the I.G.Y. is to study the effect on the earth of phenomena occurring on the surface of the sun,' which can only be even partly correct for the minority of disciplines listed.

R.M.G.

Obituary Notices

Dr. L. J. SHERMAN

Leon J. Sherman, aged 39, a meteorologist at the University of California's Los Alamos Scientific Laboratory, was killed instantly in a motoring accident on 2 March 1957. Dr. Sherman, who joined the Laboratory staff in August, 1955, received his B.A. degree in mathematics and physics in 1939, his M.A. in mathematics in 1941, and his Ph.D. in meteorology in 1950, all from the University of California at Los Angeles. Before coming to Los Alamos he was an associate professor in the Department of Meteorology at Florida State University, Tallahassee. Dr. Sherman was a member of the American Meteorological Society and the American Geophysical Union. He had been a Foreign Member of the Royal Meteorological Society since 1952.

E.A.C.

Mr. R. G. K. LEMPFERT, C.B.E., M.A.

By the death on 24 June 1957 of Mr. R. G. K. Lempfert the Society has lost a Fellow who took a very active part in its affairs over a long period before the second World War, and the Meteorological Office a retired member of the staff who in one respect was unique, for he was the first assistant with a scientific training and a university degree to be appointed to the Office.

Rudolf Gustav Karl Lempfert was born at Manchester on 7 October 1875, the son of Rudolf Bernard Lempfert. After passing through Manchester Grammar School Lempfert was admitted to Emmanuel College, Cambridge, where he came under the influence of W. N. (later Sir Napier) Shaw and after taking a first in the two parts of the Natural Sciences Tripos in 1896 and 1898 worked for a time under Shaw in the Cavendish Laboratory. Then followed four terms spent as a schoolmaster at Rugby. Meanwhile Shaw had become effective head of the Meteorological Office, where he found the peculiarity that none of the staff had had any preliminary scientific training. As a first step to rectify this position he induced his former pupil at Emmanuel to become his personal assistant in 1902. The close association between the two men continued unbroken until Shaw's death in 1945. Lempfert's appointment was abundantly justified and after helping Shaw in various scientific enquiries he was given rapid promotion, becoming Superintendent of Instruments in 1905, of Statistics in 1906 and of Forecasting in 1910. This post he held during the first World War. The experienced forecasters in the Office at that time were ageing men and the insistent call of the Royal Flying Corps, as it was then called, for night forecasts (which had not previously been prepared) meant that Lempfert himself had for a time to take on a substantial part of this work in addition to the heavy responsibility of his duties as superintendent. At this time the present writer was intimately associated with him in the forecast work. After the war, and with the merging of the Meteorological Office in the Air Ministry, Lempfert became Assistant Director, a post which he held until his retirement from the Office in 1938. For his work during the first World War he was awarded the C.B.E.

Lempfert's connection with the Society commenced in 1904 when he was elected a Fellow, becoming a member of the Council in 1908, Vice-President (1911-12 and 1932-33), Foreign Secretary (1917-29) and President (1930-31). At the time of his death he had been a Fellow for the long span of 53 years. He published numerous papers, in several cases in collaboration with others. Probably the most important was the ' Life history of surface air currents ' (Shaw and Lempfert) which appeared as a separate memoir in 1906. In this the trajectories of air particles over the earth's surface were traced in different meteorological situations and much new light was thrown on air movements in and around depressions. In particular the old conception of a depression as a more or less circular vortex of rotating air drawing in fresh supplies from all sides to supply the rising current at the centre was shown to have little relation to the facts. It was regrettable that financial stringency led to only 350 copies of this important paper being printed. It was the fortunate few only who were lucky enough to obtain a copy. At this time Lempfert also became interested in ' Line squalls ' and published two papers thereon (the second jointly with R. Corless) in the *Quarterly Journal* in 1906 and 1910. There followed in 1913 ' British weather forecasts : past and present,' a short paper packed with information about the British Synoptic Weather Service which will remain of great value to historians. Lempfert's first Presidential Address dealt with the work of the Meteorological Office at Cardington and was a tribute to the memory of the gifted young meteorologist M. A. Giblett who had then recently lost his life in the disaster to the Cardington airship, R 101. The second Address dealt with ' The presentation of meteorological data,' a matter which had forced itself on Lempfert's attention during his years as Superintendent of Statistics at the Meteorological Office. Lempfert was also the author of two books on meteorology, *Weather science*, a small popular book for the general reader, and *Meteorology*, a more substantial publication which appeared in 1920. As recently as 1955 he with the assistance of Miss E. E. Austin, published the *Selected meteorological papers of Sir Napier Shaw, F.R.S.*, in compliance with a wish of Sir Napier, expressed in his Will. Lempfert also played a major part in the establishment of another memorial to his former chief, the Napier Shaw Memorial Library in the Cavendish Laboratory at Cambridge. Outside meteorology his chief interest lay in music. He was an accomplished musician, as had been his father and mother before him. It must have been a great blow to him that when on his retirement from official work he had more time to devote to his hobby, a rheumatic affection of the arms made it impossible for him to continue playing on his beloved violin. No notice about Lempfert would be complete which did not mention his innate courtesy. No one could be in his presence for long without realising that they were in the company of a perfect gentleman.

J. S. Dines.

Professor C.-G. ROSSBY

The death of Professor Carl-Gustav Rossby, Honorary Member and Symons Gold Medallist of th Royal Meteorological Society, on 19 August 1957, in Stockholm, at the height of his influence and renown came as a grievous shock to the meteorological world and to a scientific world much wider than this. Ha his restless questing spirit allowed him to take warning from evidences of strain in recent years and to b content with leading his own Institute in Sweden he might, who can tell, have continued to give light t his profession for many years, but he had never learned how to relax. His last year was as active as any i his remarkable career; he had worked intensively in the United States, made visits on scientific business t many countries of the Old and New World, given much time and thought to the forthcoming Genera Assembly of the International Union of Geodesy and Geophysics and especially of the constituent Associa tion of Meteorology of which he was President, and, as recognition of his scientific standing, was due t preside at a meeting of a special committee set up by the International Council of Scientific Unions to surve the wide field of Antarctic scientific research. And with all this he had somehow found the time to continu with his scientific writing and at least one posthumous publication will shortly come to remind us of hi insight. If ever a man could be said to have died in harness it could be said of Rossby and at the presen time there is no one person with the reputation, vision and energy to take his place. Personal ambition an disinterested striving for worthwhile ends are rarely unrelated and Rossby enjoyed leadership, expande to his role and only with an effort could be patient with ideas which did not fit into his pattern. But hi mature years had brought to him a firm considered philosophy on the part which science in general, an meteorology in particular, must play in the modern world; they were not quiet academic roles but urgen strivings for humanistic ends. Those who were privileged to have known him in recent years were aware of a rare dynamic with a potential yet untapped; they feel not only the loss of one who had done much bu the denial of a promise of greater leadership to come, in a generation when few can measure up to the magnitude of the task.

Rossby was born in Stockholm on 28 December 1898 and graduated from the University of that city. His later studies took him to Bergen at the great period of Norwegian meteorology, and for a time to Leipzig and Lindenberg, but it was in 1926, after further work in Stockholm, that he took the decisive step in his career, a visit to the United States to join the staff of the Weather Bureau, a visit which was to last effectively for more than 20 years, to lead to American citizenship, the companionship of a delightful and understanding American wife, the joys of a growing family and a position in American meteorology second to none. From the Weather Bureau he moved to California to organize a meteorological service for civil aviation and then, in 1928, came the first major academic appointment, that of creating the new department of meteorology at the Massachusetts Institute of Technology, the first chair for Rossby being the first chair for meteorology to be promoted in the United States. That the appointment was a success is perhaps best shown by the leading position of that department at the present time. For one or two years, a decade later, his services were again secured by the Weather Bureau from where he moved again to the academic world, this time to Chicago, although his consultant work for the Government brought him into close touch with vital operations of war in several theatres. British as well as American meteorologists who experienced the impact of his visits oversea speak of his stimulating presence. After the war it was back to Chicago but not to settle ! Sweden called him once more and once more he was to pioneer a new department, an Institute of Meteorology connected with his old university but to become known as the International Institute of Meteorology and legitimately so, for funds were provided from American as well as Swedish sources and a number of inter- national bodies, including UNESCO and IUGG, were to provide a subvention; students were drawn from many countries and the international seminars held under his leadership were renowned.

Rossby's personal contributions to the scientific literature do not, in point of numbers, make up a formidable bibliography, but in point of quality and originality some of them stand very high. In the late twenties there were his early papers on atmospheric turbulence and the theory of pressure changes. In the thirties his interests continued between oceanography and meteorology as exemplified by a sequence of papers in the series of M.I.T. and Woods Hole Oceanographic Institution, e.g., ' A generalization of the theory of the mixing length,' 1932; ' The layer of frictional influence in wind and ocean currents,' 1935; ' On the momentum transfer at the sea surface,' 1936; ' Dynamics of steady ocean currents,' 1936; His growing interest in large-scale circulations is shown by papers in the *Journal of Marine Research*, 1937, 38 and 39, but perhaps his most productive thinking, in the light of later events, reached published form in 1939 and 1940 : ' Relation between variations in the intensity of the zonal circulation,' etc., *Journal of Marine Research*, 1939 and 'Planetary flow patterns,' Royal Meteorological Society, *Quarterly Journal* (*Supplement* 1940). These were to establish the value of vorticity as a parameter in the study of meteorological circulations, gave rise to the now familiar concept of Rossby Waves, and provided the basis on which, largely by his influence, electronic computing methods were in later years to be adapted to numerical integration of the barotropic equation, the first phase of dynamical weather prediction by computation which, almost 20 years later, remains one of the highlights of current meteorological progress. From this time until after his translation back to Stockholm, Rossby's main thinking was devoted to the large-scale problems, stimulated as it was by the rapid increase in observational data from the free atmosphere. A few valuable references are : ' Kinematic and hydrostatic properties of certain long waves in the Westerlies,' University of Chicago, 1942; ' On a mechanism for the release of potential energy,' *Journal of Meteorology*, 1949; ' On the dispersion of planetary waves in a barotropic atmosphere,' *Tellus*, 1949 (*Tellus* was then a new journal initiated and edited by Rossby himself); ' On the nature of the general circulation of the lower atmosphere,' in *The atmosphere of the earth and planets*, Chicago

Press, 1949. These papers are but a selection from a most productive period but Rossby was not even then content to develop his established line. In recent years, owing almost entirely to his initiative, new life was given to the study of the microchemical constituents of the atmosphere and a network of stations for the analysis of air and rainwater has been developed, mainly in Europe and America. His journal *Tellus* publishes data regularly on the content of sodium, chlorine, sulphur, potassium, calcium, and nitrogen nitrate and ammonia radicals; a paper ' On the chemical climate and its variations with the atmospheric circulation pattern,' *Tellus*, 1955, well illustrates the way his thoughts were running.

As a scientist judged in history by his original work, Rossby will hold a distinguished place, certainly in the first rank of meteorologists, but in an appraisal of his life one must return to his outstanding qualities, his leadership and vision, his organizing zeal and effectiveness, his talent for recognizing talent and above all his unsparing energy. The impetus he gave to his science will be felt for years to come.

R.C.S.

Professor H. U. SVERDRUP, Ph.D.

The death of Professor Harald Ulrik Sverdrup on 21 August 1957 was a sore blow to the science of Oceanography which he had served so long and magnificently. It came little more than a week before the opening of the 45th Statutory Meeting of the International Council for the Exploration of the Sea, of which he had been President since October 1955, and was felt all the more because that meeting took place in the city of Bergen, with which Sverdrup had such long and intimate associations. Of late years Sverdrup had taken a very keen interest in the inter-relationships between the life in the sea and the changing conditions of its milieu, and had greatly helped workers in fisheries research to understand those elements of marine physics which bear upon the production of optimum conditions for the food of fish. This particular display of interest on the part of such a distinguished marine physicist had been very warmly welcomed by the marine biologists, and had already proved extremely helpful to them. Readers who would wish to scan a detailed report on Sverdrup's accomplishments and a list of his publications up to the age of sixty may be referred to the contribution by two of his students and friends, Revelle and Munk, in the special volume of the *Journal of Marine Research*, (VII, 3, November 1948) which was issued in honour of his sixtieth anniversary. At that age, after having spent twelve years as head of the Scripps Institution of Oceanography in La Jolla, California, Sverdrup returned to Norway to become director of his native country's Polar Institute in Oslo. Of late years he had driven himself so hard that his friends had witnessed with dismay an obvious failing of health. His assumption of certain extra duties, which can rightly be deemed in the true Nansen tradition, had laid an extra burden upon him. This new work was the inspiration and organization of expert help to the fishermen of India – a task to which he applied the drive and knowledge which were so characteristic of him.

In all countries where the study of the sea is a scientific interest, the death of Sverdrup came as the loss of a friend to whom much was owed for unstinting and generous guidance. In addition to remembering him as an oceanographer of great renown, personal recollections will fix upon the numerous happy occasions when Sverdrup's cheerful personality enlivened international meetings of various kinds. He was such a happy man, and such a great asset on social occasions, that his perfect bonhomie will remain green in the memory of the host of oceanographers to whom he was both mentor and friend. That this can be said with absolute sincerity of a man who has such great accomplishments of courage and knowledge to his credit, means that he was a very remarkable man indeed. Reading Sverdrup's books one feels that they are from the pen of a man who thoroughly knows the sea. As Professor Hans Pettersson wrote when reviewing Sverdrup's work on the oceanography of the North Siberian Shelf, the work was that of ' a rare combination of an explorer, an experimenter of great skill and judgement, and an able mathematician '. British meteorologists, amongst whom many will remember Sverdrup's address on ' The circulation of the oceans ' at the joint meeting of the Royal Astronomical and Royal Meteorological Societies in London last May, will not lack acquaintance with his book ' *Oceanography for Meteorologists* ' first published in 1942; and they will certainly not need the presentation here of any list of the important papers in which Sverdrup made his contributions to the understanding of the interactions between the oceans and the atmosphere. It would be possible to fill a dozen pages of this *Journal* by merely listing Sverdrup's works, but most people will remember and admire particularly his achievements during the ' *Maud* ' Expedition when, during the two-year period from 1922 to 1924, the ship was drifting with the ice over the North Siberian Shelf. They will remember his very important findings (in association with his collaborator Finn Malmgren) about the differing inter-relationships of wind travel and ice drift as between spring and summer; they will recall the construction of a remarkable current-meter in association with Dahl; they will not forget the meticulous way in which he worked to obtain true salinity values, and they will preserve the fullest admiration for the persistence under arduous conditions with which Sverdrup pursued his studies of the very intricate hydrodynamical problems encountered in his attempt to explain the tidal phenomena occurring in the shallow waters of the shelf. These achievements were all the more remarkable for having been attained in the middle of the Arctic ice with the strictly limited resources of reference afforded by a research ship's library.

Seven years after the *Maud* Expedition, Sverdrup was again battling with Arctic ice – this time in the submarine *Nautilus* during Sir Hubert Wilkins's attempt to reach the North Pole under the ice. Though the goal was not achieved, dives were made underneath the ice and valuable information gained before, on account of damage sustained, the submersible returned to Norway to be scuttled later off Bergen.

It would indeed be a long notice which would do proper justice to the memory of H. U. Sverdrup, but, whatever its length, its main import would be what we have briefly attempted to convey – that the world of oceanography has lost a prince among men; a man in whom the highest professional competence was united with compelling attributes of charm, friendliness, and personal courage.

Professor Sverdrup was elected into Honorary Membership of the Royal Meteorological Society on 20 March 1935.

J.N.C.

Dr. A. W. M. BATTERSBY, L.R.C.P., L.R.C.S. (Edin.), D.P.H.

Dr. Alexander Watson Muir Battersby, one of the few medical practitioners to be numbered among the Society's recent membership, died on 23 September 1957 at the age of 54. He was educated at the Academy and University of Glasgow, his native city, and trained for his profession at St. Mungo's College there, becoming qualified in 1927. Although for a time in private practice with his father, Dr. Battersby gave the greater part of his life to public or national ministration. On two occasions he voyaged as ship's surgeon. He had also been resident house surgeon, then senior resident medical officer, at Glasgow Royal Infirmary, and physician superintendent at Lightburn Fever Hospital, Shettleston. His later appointments brought him south of the Border. From 1938 to 1950 he was both assistant county medical officer and assistant school medical officer for Shropshire. Subsequently he held the post of area medical officer for Uttoxeter. Meteorology had long been one of Dr. Battersby's great interests, but the heavy calls of his profession did not allow him to devote as much time to the study of our science as he could have wished. His Fellowship in the Society dated from 15 October 1947.

E.L.H.

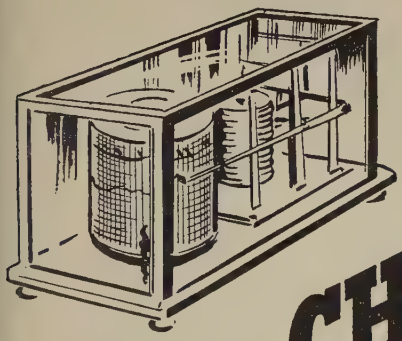
THE PHYSICAL SOCIETY EXHIBITION OF SCIENTIFIC INSTRUMENTS AND APPARATUS

Monday to Thursday, 24 to 27 March: The 42nd Physical Society Exhibition will be held at the Halls of the Royal Horticultural Society, Westminster, S.W.1.

This Exhibition is a unique one in that emphasis is placed on new developments in scientific instruments and apparatus and on the possibility of their applications in novel ways.

The opening ceremony will be performed in the New Hall by Prof. N. F. Mott, F.R.S., on 24 March, at 11 a.m. Discourses will be given at 5.45 p.m. as follows: 24 March, ' Goethe's colour experiments,' by Mr. M. H. Wilson; 25 March, ' Some uses of physics in archeology,' by Dr. E. T. Hall; and 26 March, ' Some recent investigations with aircraft on the physics of clouds and rain,' by Mr. R. J. Murgatroyd.

The Royal Meteorological Society will again be taking a small stand to display its publications. The number of the stand will be 119, and it is hoped to welcome there members and readers of the *Quarterly Journal.* Admission will be free, but by ticket only. A limited number of tickets is available on written application to the Society.



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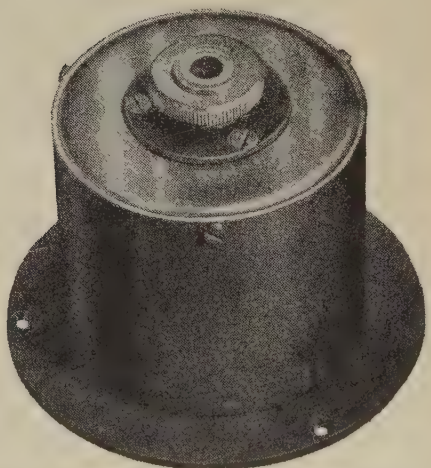

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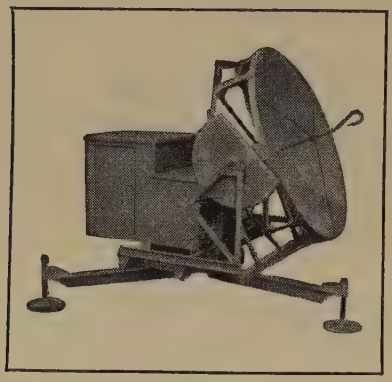

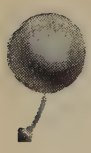

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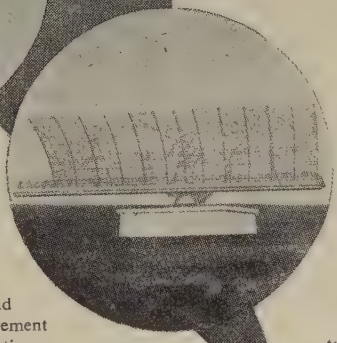

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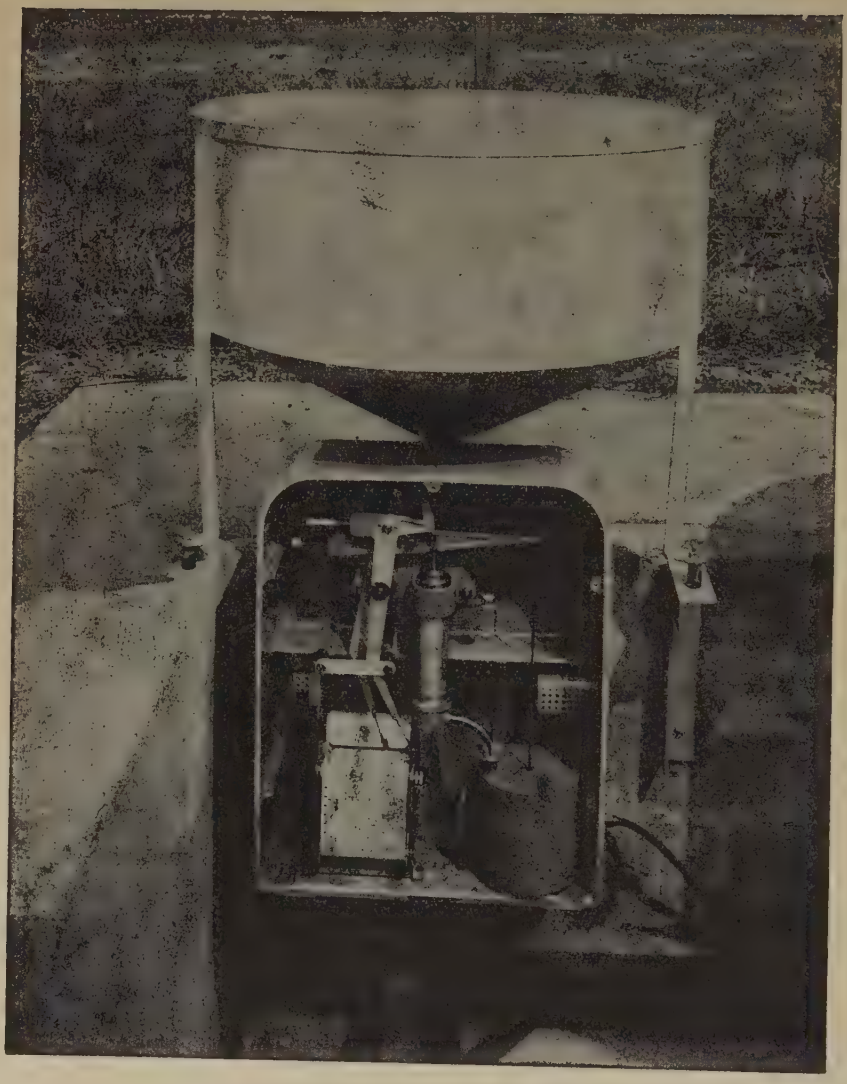

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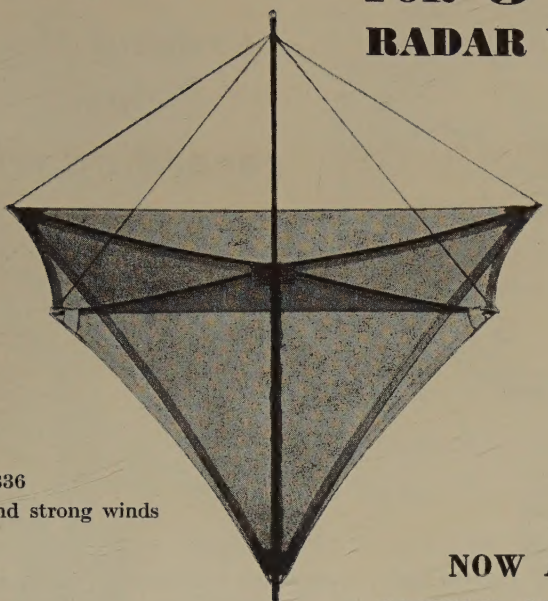

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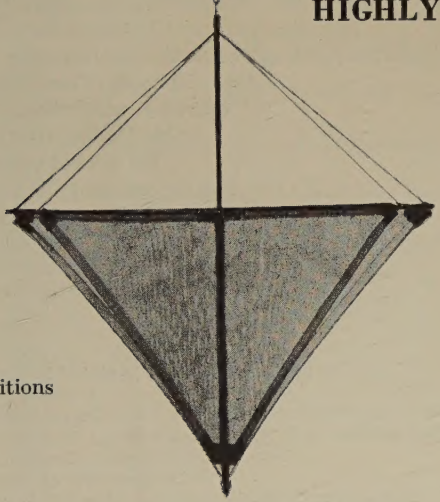

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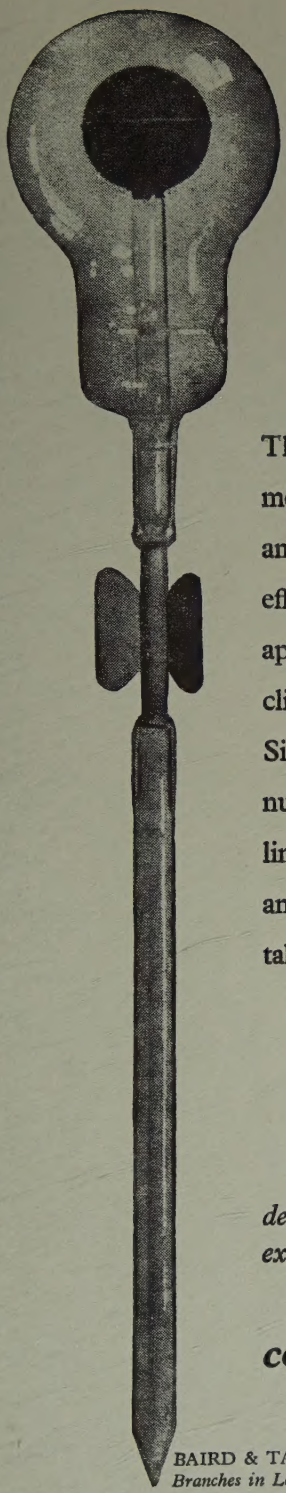

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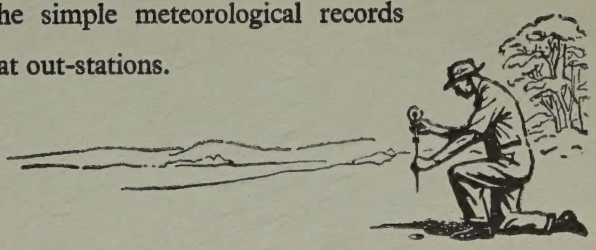

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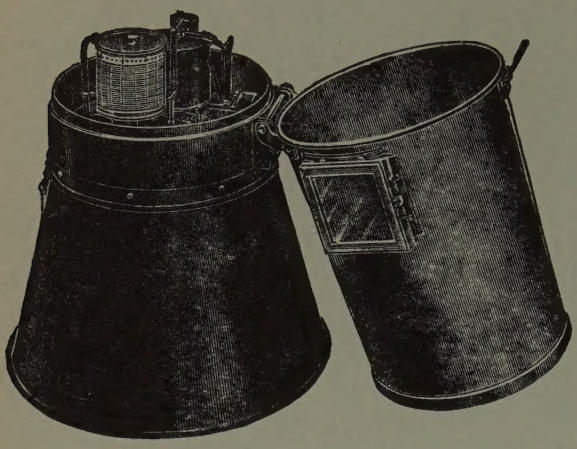

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